

Fig. 10. Predicted dependence of the location of the Fermi energy with respect to the saddle point, μ_f , on T_c (both quantities are measured in K) [18].

This definiteness is no surprise, since the basis of these calculations is the experimental dependence $\alpha(T_c)$ for definite compounds.

In the previous derivation it was assumed that both, μ_f and ω_0 , are much larger than T_c . This happens not to be the case for the largest T_c ($\mu_f = T_c$ is presented by the dashed line at Fig. 10). Of course this can be corrected (actually already for $T_c = 90\text{K}$ the correction is small), however this region is suspicious in the sense that the small value of the isotope shift is likely not entirely defined by the cut-off of the integrals, and the phonon frequency can enter the interaction, reducing the value of α . Otherwise small values of α require large values of x (see eq.(35)), and hence unphysically large values of ω_0 ; see Fig. 11 representing the dependence $\omega_0(T_c)$ obtained from $x(T_c)$, $\mu_f(T_c)$ and $x = \omega_0/\mu_f$.

Fig. 11. The variation of the effective phonon frequency ω_0 with T_c (both in K) [18].

The fact that the isotope shift for $\text{Cu}^{63}\text{-Cu}^{65}$ is negative and increases in magnitude with increasing T_c (see ref. [26]), can also be considered as evidence for inapplicability of the theory, based on the assumption of the purely cut-off nature of the isotope shift, to substances with extreme T_c . On