We see from the above example that a strong magnetic field can "trap" electrons so that they can move only so far in either the r or the z directions. There is one particular field which perhaps merits individual attention; a charge-free region in which the electric potential is given by

$$V = \frac{\omega_0^2}{2\eta}\left(\frac{r^2}{2} - z^2\right) + V_0 \qquad (4.29)$$

This field can be produced by hyperbolic electrodes indicated in cross-section in Fig. 4.7. Suppose we have also a uniform magnetic field of strength B in the z direction. There can be no magnetic forces in the z direction, so the equation of motion in this direction is

$$\ddot{z} = -\omega_0^2 z \qquad (4.30)$$

This implies a sinusoidal oscillation of radian frequency ω_0. If it were not for the magnetic field, electrons would escape in the r direction. We see that, remote from the axis, the maximum potential (that at $z = 0$) is closely given by

$$V = \frac{\omega_0^2 r^2}{4\eta} \qquad (4.31)$$

From (4.28), remote from the axis the cutoff potential approaches

$$V = \frac{\omega_c^2 r^2}{8\eta} \qquad (4.32)$$

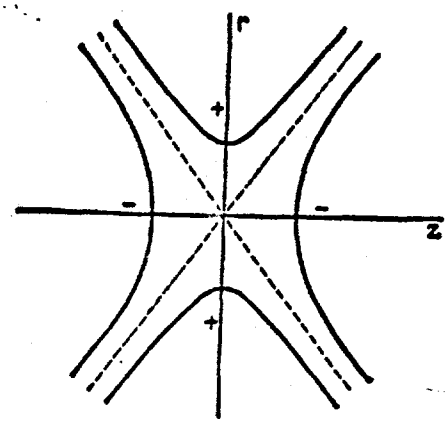

FIG. 4.7—Electron motion between hyperbolic electrodes may be limited to a certain region by use of an axial magnetic field.

Thus, the condition under which electrons cannot get indefinitely far away from the axis is that

$$\omega_c > \sqrt{2}\omega_0 \qquad (4.33)$$

We see that it is possible to obtain a pure sinusoidal motion of electrons trapped in this combination of electric and magnetic fields.