Unless the vacuum energy density is very large, $M/\sqrt{N} \gtrsim 10^{16}$ GeV, this condition is not very interesting. Moreover, it will be superseded by the next constraint.

Second, to ensure that the fluctuations in the axion density do not lead to excessive fluctuations in the temperature of the CMBR, the axion mass must satisfy:

$$m_a \lesssim 10^{-2} \left(\frac{N}{M_{14}^2}\right)^{2.4} \text{ eV}. \tag{7b}$$

For $M/\sqrt{N} \gtrsim 7 \times 10^{14}$ GeV, this condition implies that the axion mass *must* be less than about 10^{-6} eV. In chaotic inflation[13] the value of M is about 8×10^{15} GeV, implying that the axion mass must be *less* than about 10^{-11} eV, corresponding to a symmetry breaking scale greater than 10^{18} GeV.

These limits are general. If we assume that axions do in fact provide closure density, then to avoid excessive CMBR anisotropies we must require the stronger condition

$$m_a \lesssim 10^{-8} \left(\frac{N}{M_{14}^2}\right)^{2.4} \text{ eV}. \tag{7c}$$

This is perhaps the most stringent and interesting condition. In chaotic inflation it precludes the axion from being the dark matter. If $(N/M_{14}^2)^{2.4} \gtrsim 1$, one must require a very small axion mass, and correspondingly a very small misalignment angle, to achieve closure density in axions. (And of course an even smaller and less likely angle, to achieve a smaller density!) Current[14] and proposed[15] cosmic axion searches have focused on the mass interval 10^{-6} eV to 10^{-4} eV. To the extent our considerations make the low mass axion plus inflation option less attractive they further emphasize the importance of this accessible interval, which is motivated by the "no inflation after PQ breaking" case.

Let us summarize. In the absence of inflation, or if the PQ transition occurs after inflation, axions are cosmologically overproduced unless the axion mass is more than about 10^{-6} eV. If the Universe underwent inflation after the PQ transition it might seem that a very low mass axion is only improbable and that closure density of axions could be achieved for any value of the axion mass less than about 10^{-4} eV, provided that the misalignment angle is sufficiently small.[8] However, here we have shown that when one takes into account the effect of the quantum fluctuations in the axion field that arise during inflation—and the resulting CMBR temperature fluctuations—closure density of axions is only consistent for values of the axion mass *less* than about $10^{-8}\,(N/M_{14}^2)^{2.4}$ eV. Based upon the isotropy of the CMBR alone, it follows that in the inflationary case the axion mass must be less than about $10^{-2}\,(N/M_{14}^2)^{2.4}$ eV, whether or not axions provide closure density.

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