same magnitude, leading to fluctuations in the temperature of the cosmic microwave background radiation (CMBR). Isocurvature axion fluctuations have been discussed elsewhere in detail;[10] we will now review the relevant points.

For simplicity, suppose that PQ symmetry is broken when a complex scalar field $\vec{\phi}$ which carries PQ charge takes on a vacuum expectation value $\langle\vec{\phi}\rangle = f_a \exp(i\theta)/\sqrt{2}$, which minimizes its scalar potential $V(\vec{\phi}) = \lambda(|\vec{\phi}|^2 - f_a^2/2)^2$. The flat, θ degree of freedom corresponds to the axion. In de Sitter space all minimally coupled, massless scalar fields have quantum fluctuations characterized by amplitude $H_I/2\pi$ where H_I is the expansion rate during inflation. For the axion field these fluctuations lead to fluctuations in θ:

$$\delta\theta = \frac{H_I}{2\sqrt{2}\pi f_a} \simeq 4 \times 10^{-5} \left(\frac{M_{14}^2}{N}\right) m_{-6}, \tag{4}$$

where $m_{-6} \equiv m_a/10^{-6}\,\mathrm{eV}$, the value of the Hubble constant during inflation has been written as $H_I^2 = 8\pi M^4/3m_{\rm Pl}^2$, M^4 is the false-vacuum energy, and $M_{14} \equiv M/10^{14}\,\mathrm{GeV}$.[11]

Fluctuations in the axion angle θ lead to constraints to the possible values of the misalignment angle θ_1. First, it is not possible for θ_1 to be smaller than the fluctuations in θ; thus

$$\theta_1 \gtrsim \delta\theta \simeq 4 \times 10^{-5} \left(\frac{M_{14}^2}{N}\right) m_{-6}. \tag{5}$$

Next, fluctuations in the misalignment angle ultimately result in density perturbations of the same amplitude, which, on the largest angular scales ($\gg 1°$), lead to anisotropies of the CMBR temperature. For $\Omega_a \simeq 1$, $\delta T/T \sim \delta\rho_a/\rho_a \sim \delta\theta/\theta_1$. If Ω_a is smaller than unity, i.e., if some species other than axions provides closure density, then the magnitude of this effect is reduced by a factor Ω_a: $\delta T/T \simeq \Omega_a(\delta\theta/\theta_1)$. On large angular scales the CMBR temperature is remarkably smooth, as evidenced by the absence of a quadrupole anisotropy at the level,[12] $\delta T/T \lesssim 3 \times 10^{-5}$. This results in the following additional constraint to the initial misalignment angle θ_1:

$$\theta_1 \lesssim 0.1 \left(\frac{N}{M_{14}^2}\right) m_{-6}^{0.18}. \tag{6}$$

The three constraints to the initial misalignment angle θ_1—Eqs. (3, 5, 6)—based upon the conditions $\Omega_a h^2 \lesssim 1$, $\theta_1 \gtrsim \delta\theta$, and $\delta T/T \lesssim 3 \times 10^{-5}$ are illustrated in Fig. 1. They imply constraints upon the axion mass. First, to ensure that fluctuations in the misalignment angle do not lead to overproduction of axions the axion mass must satisfy:

$$m_a \lesssim 3 \times 10^4 \left(\frac{N}{M_{14}^2}\right)^{2.4} \mathrm{eV}. \tag{7a}$$