Since the initial misalignment angle θ_1 is a random variable, at the time of PQ symmetry breaking the value of θ_1 will be different and uncorrelated in different causally distinct regions of the Universe. In the absence of inflation, these different regions are very small, and today the Universe is comprised of a very large number of regions that each had a different value of θ_1. To obtain the average axion energy density, one uses the *rms* average of θ_1, which is just $\pi/3$, in Eq. (2). In this circumstance the limit to $\Omega_a h^2$ based upon the age of the Universe,[7] $\Omega_a h^2 \lesssim 1$, implies a lower bound to the axion mass: $m_a \gtrsim 10^{-6}\,\mathrm{eV}$, or an upper bound to f_a: $f_a/N \lesssim 10^{13}\,\mathrm{GeV}$. The upper bound to f_a is significantly below two interesting energy scales: the unification scale of $10^{14}\,\mathrm{GeV}$ to $10^{16}\,\mathrm{GeV}$ and the Planck scale, a somewhat disappointing fact. (However there are models, particularly those involving supersymmetry breaking in a "hidden sector", where the geometric mean of the weak symmetry breaking scale and the Planck mass, $\sqrt{m_{\mathrm{Pl}} m_{\mathrm{weak}}} \approx 10^{11}\,\mathrm{GeV}$ is the fundamental symmetry-breaking scale.)

If the Universe inflated before or during PQ symmetry breaking the fluctuations in the axion field take an entirely different form. While the average of θ_1^2 over many causally separate volumes is still $\pi/3$, the practical relevance of this fact is nil, because the entire presently observable Universe lies within one causal region where θ_1 is constant.

A number of authors[8] have pointed out that axion masses smaller than $10^{-6}\,\mathrm{eV}$ (PQ symmetry breaking scales greater than $10^{13}\,\mathrm{GeV}$) could be consistent with $\Omega_a h^2 \lesssim 1$, provided that θ_1 was sufficiently small:

$$\theta_1 \lesssim (m_a/10^{-6}\,\mathrm{eV})^{0.59}. \tag{3}$$

In this case, then, we would be living in a rare, axion-poor region of the Universe. If the Universe did indeed undergo inflation, the fundamental laws of physics do not determine θ_1. Despite its cosmic import the local value of this parameter is an "historical accident," and can only be determined through direct measurement of $\Omega_a h^2$ and m_a.

One might then be left with the impression that if the Universe underwent inflation any value of f_a can be tolerated, provided that θ_1 is appropriately small. Values of $f_a/N \gtrsim 10^{13}\,\mathrm{GeV}$ are only improbable, not forbidden. The main purpose of this *Letter* is to point out that additional, very important constraints emerge when fluctuations in the axion field that arise during inflation are taken into account.

Quantum fluctuations in the axion field have two important effects: (i) They set an absolute minimum to θ_1, the effective misalignment angle to be used in the energy estimate Eq. (2)—it can be no smaller than the size of its fluctuations; and (ii) They give rise to isocurvature axion perturbations,[9] i.e., fluctuations in the local axion-to-photon ratio. At late times ($t \gg t_{EQ} \simeq 4.4 \times 10^{10} h^{-4}\,\mathrm{sec}$), these evolve into density perturbations of the