because this field modulates the phase of the instanton amplitude. At low temperatures the free energy has a maximum value of about $\Lambda_{\rm QCD}^4$, is periodic in the "axion angle" $\theta \equiv a/(f_a/N)$, and is minimized at a value of $\theta = 0$. The mass of the axion is determined by the curvature of the free energy at $\theta = 0$ and is given approximately by Eq. (1). At high temperatures instanton effects are strongly suppressed, and for $T \gg \Lambda_{\rm QCD}$ the free energy is essentially independent of the axion field. Thus, when PQ symmetry breaking occurs ($T \sim f_a$), no value of the axion angle is singled out dynamically, and one expects that the value of the axion angle in different causally distinct regions will be randomly distributed between $-\pi$ and π. Averaging will occur over a distance scale corresponding to the horizon size at the Peccei-Quinn transition, which is negligibly small cosmologically. Thus the primæval energy density associated with the misalignment of the axion field should be of order $\Lambda_{\rm QCD}^4$, and close to uniform. Further, when the axion mass exceeds $3H$ ($H \sim T^2/m_{\rm Pl}$ is the expansion rate and $m_{\rm Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass) the axion field will begin to relax toward $\theta = 0$. Because it has no efficient way to shed energy, the field is left oscillating. The energy density in oscillations of the axion field behaves as nonrelativistic matter during the subsequent evolution of the Universe, and may be interpreted in particle language as a gas of zero-momentum axions. These relic axions, if present, behave like cold dark matter and are a promising dark matter candidate.

The contribution of these axions to the present mass density of the Universe is estimated to be[3]

$$\Omega_a h^2 \simeq 0.13 \times 10^{\pm 0.4} \Lambda_{200}^{-0.7} f(\theta_1^2)\theta_1^2 (m/10^{-5}\,{\rm eV})^{-1.18}, \tag{2}$$

where Ω_a is the fraction of critical density contributed by axions, the present value of the Hubble constant $H_0 = 100h\,{\rm km\,sec^{-1}\,Mpc^{-1}}$, $\Lambda_{\rm QCD} = \Lambda_{200} 200\,{\rm MeV}$, and θ_1 is the initial misalignment angle. The function $f(\theta_1^2)$ accounts for anharmonic effects, and is of order unity (and specifically $f \to 1$ for $\theta_1 \ll 1$). The $10^{\pm 0.4}$ factor is an estimate of theoretical uncertainties—e.g., in the temperature dependence of the axion mass. Other possible physical process which may affect the result are discussed in Refs. 6. Note that closure density in axions is achieved for a mass somewhere between 10^{-6} and 10^{-4} eV.

The unusual dependence of the axion energy density upon the axion mass is easily understood. Regardless of the value of the axion mass, the energy density associated with the initial misalignment of the axion field is of order $\Lambda_{\rm QCD}^4$; once the axion field starts to oscillate that energy density red shifts as R^{-3} (R is the cosmic scale factor). The axion field begins to oscillate when the axion mass $m_a(T) \simeq 3H$: For smaller masses the axion oscillations begin later, and the energy density trapped in the misalignment of the axion field is diminished less.