Peccei-Quinn (PQ) symmetry with its attendant pseudo-Nambu-Goldstone boson—the axion—remains the most attractive and promising solution to the strong-CP problem.[1] Moreover, the axion arises naturally in supersymmetric and superstring models. The axion mass and PQ symmetry breaking scale are related by

$$m_a \simeq \frac{\sqrt{z}}{1+z}\frac{f_\pi m_\pi}{(f_a/N)} \simeq \frac{0.62\,\mathrm{eV}}{(f_a/N)/10^7\,\mathrm{GeV}}, \tag{1}$$

where f_a is the PQ symmetry breaking scale, $z \simeq 0.56$ is the ratio of the up to down quark masses, f_π and m_π are pion decay constant and mass, and N is the color anomaly of PQ symmetry. At present there is little theoretical guidance as to the key parameter: the PQ symmetry breaking scale f_a (or equivalently the axion mass).

Relic axions arise due to three distinct cosmological mechanisms: (i) thermal production[2]—for an axion of mass greater than about 10^{-4} eV axions thermalize shortly after the QCD transition and, today, like neutrinos, should have a relic abundance of order $30\,\mathrm{cm}^{-3}$; (ii) the "misalignment" mechanism[3]—at high temperatures the dependence of the free energy upon the axion field becomes very weak and thus the value of the axion field will in general not coincide with the low-temperature, CP-conserving minimum; due to this misalignment the axion field is set in motion around the time of the QCD transition; and (iii) axionic string decay[4]—since PQ symmetry breaking involves the spontaneous breakdown of a global $U(1)$ symmetry, strings are produced; they decay by radiating (among other things) axions. While the thermal population of axions dominates for axion masses greater than about 10^{-2} eV, there are strong constraints on axions in this parameter regime due to the potential observability of their decays and to the effect they would have upon the evolution of red giant stars.[1] Thermal axions can contribute at most 10% of critical density. (Relic thermal axions of mass 3 to 8 eV have not been ruled out and recently a search for them has been performed[5].)

For axion masses greater than about 10^{-2} eV misalignment and axionic string decay are the dominant production processes, and sufficient numbers of axions can be produced to provide closure density. The importance of axionic string decay is still a matter of intense debate. It seems to be agreed that axion production through this mechanism is somewhere between being comparable to and about 100 times more important than the misalignment mechanism[4]. Here we are mainly interested in the case where the Universe inflated either before or during PQ symmetry breaking. Since axionic strings do not survive a period of inflation, our considerations are largely independent of this issue.

Let us briefly recall the misalignment mechanism of axion production, ignoring at first the possibility of inflation. The free energy of the vacuum depends upon the axion field only