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Electromagnetic Effects Due to Spin-Orbit Coupling

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The existence of strong spin-orbit coupling in the single-particle model of the nucleus implies the existence of a term

$$-f(r)\,(\vec{\sigma}\,\vec{\ell}) = -f(r)(\vec{\sigma}\cdot[\vec{r}\times\vec{p}]) \qquad (1)$$

in the single-particle Hamiltonian. This gives rise to an interaction of charged nucleons with external electromagnetic fields with vector potentials A. Replacing the momentum in the Hamiltonian by $\vec{p} - \frac{e}{c}\vec{A}$, we obtain from the spin-orbit coupling term the interaction energy:

$$\frac{e}{c}\,f(r)\,.\,(\vec{\sigma}\,[\vec{r}\times\vec{A}]). \qquad (2)$$

One consequence of this effect is that the magnetic moments of odd-proton nuclei should deviate from the Schmidt lines.[1] In a constant magnetic field with $\vec{A} = \frac{1}{2}[\vec{H}\times\vec{r}]$ the interaction term (2) becomes:

$$\frac{e}{2c}\,f(r)\,(\vec{\sigma}[\vec{r}\times[\vec{H}\times\vec{r}]]) = \frac{e}{2c}\,f(r)\,\left\{r^2\,(\vec{\sigma}\cdot\vec{H}) - (\vec{r}\,\vec{\sigma})(\vec{r}\cdot\vec{H})\right\} \qquad (3)$$

For a state with orbital angular momentum ℓ, total angular momentum $j = \ell \pm \frac{1}{2}$, and magnetic quantum number m_j the expectation value of the interaction energy is:

$$\pm\,\frac{e}{2c}\cdot\overline{r^2\,f(r)}\cdot\frac{2\,j+1}{2\,j+2}\,\frac{m_j}{j}\cdot H \qquad (4)$$

with the positive sign for $j = \ell + \frac{1}{2}$, the negative sign for $j = \ell - \frac{1}{2}$. The factor in front of H, for $m_j = j$, is the negative additional magnetic moment. The effect consequently places the magnetic moments inbetween the Schmidt lines, as is experimentally observed. Taking the spin-orbit splitting energy, $\overline{f(r)}(2\ell+1)\cdot\hbar$, to be about 2 Mev at $\ell = 4$, A = 100,

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1. The fact that velocity dependent two-body forces give rise to additional moments has been pointed out by Blanchard, Avery, and Sachs, Phys. Rev. 78, 292 (1950).