If the reacting material is surrounded by an infinite sphere of hydrogenous material, say water, the critical mass cannot be calculated with the help of diffusion theory because the critical radii become smaller than the mean free path. The critical radius R can be estimated as follows: We consider a fission taking place on the surface of a sphere. It creates w fast neutrons and w' slow ones. If these neutrons penetrate into the water without causing further fission, the number of neutrons returning will be $w\gamma_f + w'\gamma_s$. Here γ_f and γ_s are the albedos for fast and slow neutrons. All the returning neutrons cause fission. If, therefore,

$$w\gamma_f + w'\gamma_s \geq 1 \tag{12}$$

then the chain reaction will be self-supporting. For γ_f and γ_s we find the approximate expressions

$$\gamma_f = \frac{0.5}{1+4/R}, \quad \gamma_s = \frac{0.7}{1+1/R}$$

where R is the radius of sphere measured in centimeters. Eq. (12) has to be corrected for fissions and slowing down processes taking place in the interior of the sphere. This leads to the corrected expression

$$\begin{aligned} w\gamma_f + w'\gamma_s = &+ \frac{w}{2} RN\alpha \left[\sigma_{ff}\right] \left[(w-1)\sigma_f + w'\gamma_s\right] \\ &+ \frac{w'}{2} RN\alpha \sigma_{sf} \left[w\gamma_f + (w'-1)\gamma_s\right] \\ &+ \frac{w}{2} RN\beta \sigma^i(\gamma_s - \gamma_f) = 1 \end{aligned} \tag{13}$$

where N is the number of nuclei per cm^3; $\sigma\beta$ are two factors which take into account the fact that the neutrons are scattered within the sphere. We have used 1.5 for α and 1.3 for β as very approximate values. The values are very approximate but they occur only in the small correction terms. σ_{ff} and σ_{sf} are the cross sections for fast and slow fission per molecule respectively. σ^i is the cross section for inelastic scattering. The correction terms are small, hence the critical radius is to a good approximation independent of the dilution factor. This approximation is valid to the extent that an incident neutron may be considered to produce fission with certainty. If the dilution becomes too great (x = 0.1) this will no longer be so. In this region, the calculation will be further complicated by the resonance absorption of ordinary uranium. It is certain that in the latter region more absorbing material will be needed than for high isotopic concentrations.

In the case of UF_6, the correction term turns out to be .094, the critical radius 6.75 cm, the critical mass 6.05 kg.

A water or paraffin layer of 10 cm thickness produces about the same effect as an infinite hydrogenous reflector. Thinner layers produce smaller effects; very thin layers produce effects which tend to zero more rapidly than the thickness of the layer.* Thus, the effect of very thin

* This is due to the fact that protons give no back-scattering. The effect of the non-hydrogenous material in the layer gives of course, an effect proportional to the thickness of the layer.