Method of Calculation:

To obtain the critical amount of reacting materials, a differential diffusion theory was employed. A pair of coupled differential equations were constructed, one expressing the balance between the diffusion of "fast" neutrons from a point in the reacting material, the net number of "fast" neutrons created there by fission processes caused by both "fast" and "slow" neutrons, and the "fast" neutrons lost by inelastic scattering. The other expressed the balance between the gains and losses at a point of the "slow" neutrons. Here the losses are through fission giving "fast" neutrons and through resonance absorption by 28. The results for spherically symmetrical "fast" and "slow" neutron distribution (n_f and n_s) within the reacting material were, respectively:

$$n_s = \frac{1}{v_s r} \left[a' \sin k'r + a'' \sinh k''r\right] \tag{1a}$$

$$n_f = \frac{1}{v_f r} \left[\alpha' a' \sin k'r + \alpha'' a'' \sinh k''r\right] \tag{1b}$$

where v_s and v_f are the "slow" and "fast" neutron velocities. a' and a" are arbitrary constants to be determined through the application of boundary conditions. k', k", and a' and a" are quantities determined by the physical constants of the nuclei involved. One sees that k' is a "wave number" for the "fundamental" sinusoidal component of each distribution. The hyperbolic terms are needed to adjust the distributions to the surface conditions imposed, and since k" is always considerably larger than k', these terms suffer rapid exponential decay with distance from the surface. They thus earn the names "transients" and "surface corrections."

Specifically,

$$k'^2 = 1/2\ (A+D) + \sqrt{1/4\ (A-D)^2 + BC} \tag{2a}$$

$$k''^2 = -1/2\ (A+D) + \sqrt{1/4\ (A-D)^2 + BC} \tag{2b}$$

$$\alpha' = \frac{k'^2 - D}{C} \tag{3a}$$

$$\alpha'' = \frac{-k''^2 - D}{C} \tag{3b}$$

where

$$\left.\begin{aligned}
A &= -3\sigma_{ft}\left[\left\{\sigma_8^i - \sigma_{f8}(w-1)\right\}(1-x) + \left\{\sigma_5^i - \sigma_{f5}(w-1)\right\}x\right]N^2 \\
B &= 3\sigma_{ft}\left[\sigma_{s5}\, wx\right]N^2 \\
C &= 3\sigma_{st}\left[\left(\sigma_{f8}\, w' + \sigma_8^i\right)(1-x) + \left(\sigma_{f5}\, w' + \sigma_5^i\right)x\right]N^2 \\
D &= -3\sigma_{st}\left[\sigma_{s5}(1-w')\,x + \sigma_r(1-x)\right]N^2
\end{aligned}\right\} \tag{4}$$

Here x is the fraction of the U nuclei of the isotope 25. It is immediately apparent that A and D are negative and so k" is greater than k'. The quantity