$$A = [I\nu/(1+f)] \sum_{i=1}^{3} [F_i/G_0(-\alpha_i)] [(\alpha_i^2 - k_0\alpha_i \cot k_0(a+2x_0))](k_0^2+\alpha_i^2)^{-1}$$
$$+ \cos(k_0x_0 + \delta_i) \sin\delta_i \operatorname{cosec} k_0(a+2x_0)\, e^{-\alpha_i(a+x_0)} \qquad (46)$$

where $\delta_i = \tan^{-1}(\alpha_i/k_0)$.

One may neglect the terms containing $e^{-\alpha(a+x_0)}$ as factors because, for all cases treated, α_i has values sufficiently large to make these terms negligibly small compared to

$$(\alpha_i^2 - k_0\alpha_i \cot k_0(a+2x_0)(k_0^2+\alpha_i^2)^{-1}$$

Inserting A into Eq. 39, neglecting the last terms of Eq. 46, and setting $k = 0$, one obtains:

$$F(o) = -[(I\nu)/(1+f)] \sum_{i=1}^{3} F_i[F_0(o)/G_0(-\alpha_i)][k_0^2 + k_0\alpha_i \cot k_0(a+2x_0)]$$
$$= (k_0^2+\alpha_i^2)^{-1} \qquad (47)$$

From Eqs. 35 and 36 one obtains the ratio:

$$F_0(o)/G_0(\alpha_i) = e^{\alpha_i A_i}/f \qquad (48)$$

Here $A_i = (1/n)\int_0^1 ds\, T_c/(1+\alpha_i S)$ is the expression tabulated in Table II of (LA-258).* Upon inserting F(o), given by Eqs. 47 and 48 into Eq. 34, one obtains the current density of fast neutrons into the water tamper,

$$I = [(I\nu)/(1+f)]\{F_s - \sum_{i=1}^{3} [F_i e^{\alpha_i A_i}/(k_0^2+\alpha_i^2)][K_0^2 + k_0\alpha \cot k_0(a+2x_0)]\} \qquad (49)$$

The relation (49) enables one to compute the critical slab thickness "d" in cm. as a function of ν, σ, f, α_i, α_2, α_3, F_1, F_2, and F_3.

$$\cot k_0(\alpha d + 2x_0) = \frac{F_2 - (1+f)/\nu - k_0^2 \sum_{i=1}^{3} F_i e^{\alpha_i A_i}/(k_0^2+\alpha_i^2)}{k_0 \sum_{i=1}^{3} F_i \alpha_i e^{\alpha_i A_i}/(k_0^2+\alpha_i^2)} \qquad (50)$$

where $k_0/\tan^{-1} k_0 = 1 + f > 1$, and $x_0 = 0.71/(1+f)$.

If the active material is just critical in an infinite amount, no water being present, ($f = 0$) a finite critical thickness is possible for

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* The notation used in (LA-258) identifies $1 + f$ as c, and α_i as k. S. Frankel has had Table II extended to the much larger values of $k = \alpha_i$ required for this problem. We are greatly indebted to Frankel for suggesting the method outlined in this section.

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