$$F(k) = A\, F_0(k) - \frac{I\,\nu}{1+f} \sum_{i=1}^{3} \frac{F_i\, \alpha_i\, F_0(k)}{(k+\alpha_i)\, G_0(-\alpha_i)} \tag{39}$$

Here A is a constant determining the relative amount of homogeneous and particular solution. A is to be chosen in such a manner as to satisfy the condition, $f(a + x_0) = 0$.

By taking the inverse Laplace transform of Eq. 39 one obtains the general solution

$$f(x) = A\, f_0(x) - \frac{I\,\nu}{1+F} \sum_{i=1}^{3} \frac{F_i\, \alpha_i}{G_0(-\alpha_i)}\, f_i(x) \tag{40}$$

Here we shall make use of the asympototic expressions for $f_0(x)$, $f_i(x)$ and their corresponding Laplace transforms.

$$F_0(k) = \int_0^\infty dx\, e^{-kx}\, f_0(x) \approx \int_0^\infty dx\, e^{-kx} \sin K_0\,(x + x_0) \tag{41}$$

$$F_0(k)/(k + \alpha_i) = \int_0^\infty dx\, e^{-kx}\, f_i(x) \tag{42}$$

Multiplying Eq. 42 by $k + \alpha_i$ and integrating the first term of the right member by parts, one obtains:

$$\begin{aligned} F_0(k) &= \int_0^\infty dx\, k e^{-kx}\, f_i(x) + \int_0^\infty dx\, \alpha_i\, e^{-kx}\, f_i(x) \\ &= f_i(0) + \int_0^\infty dx\, e^{-kx}\, (d/dx + \alpha_i)\, f_i(x) \end{aligned} \tag{43}$$

By equating Eqs. 43 and 41 one obtains the following differential equation for $f_i(x)$:

$$(d/dx + \alpha_i)\, f_i(x) = \sin k_0(x + x_0), \quad f_i(o) = 0 \tag{44}$$

The solution of Eq. 44 is simply

$$\begin{aligned} f_i(x) = (k_0^2 + \alpha_i^2)^{-1} \{&\alpha_i \sin k_0(x + x_0) - k_0 \cos k_0(x + x_0) \\ &+ (k_0 \cos k_0 x_0 - \alpha_i \sin k_0 x_0)\, e^{-\alpha_i x}\} \end{aligned} \tag{45}$$

Inserting Eq. 45 and $f_0(x) = \sin k_0(x + x_0)$ into Eq. 40 one evaluates the constant A by setting $f\,(a + x_0) = 0$.

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