If one defines two functions f(x) and g(x) as follows:

$$n(x) = f(x) + g(x) \begin{cases} f(x) = 0 & \text{for} \quad x < 0 \\ g(x) = 0 & \text{for} \quad x \geq 0 \end{cases} \tag{28}$$

Eq. 25 may be rewritten as

$$f(x) + g(x) = (1 + f) \int_{-\infty}^{+\infty} dx' \, [E_1|x - x'|/2] \, f(x') + \phi(x)/(1 + f) \tag{29}$$

with the condition that $f(a + x_0) = 0$.

We shall outline the procedure used in solving the above inhomogeneous Weiner-Hopf-type integral equation for the current density given by Eq. 26. Let us define the following Laplace transforms:

$$\begin{aligned} F(k) &= \int_0^{+\infty} dx \, f(x) \, e^{-kx} = \int_0^{\infty} dx \, n(x) \, e^{-kx} \\ G(k) &= \int_{-\infty}^{+\infty} dx \, g(x) \, e^{-kx} = \int_{-\infty}^{0} dx \, n(x) \, e^{-kx} \\ F_1(k) &= \int_{-\infty}^{+\infty} dx \, \phi(x) \, e^{-kx} = \int_0^{\infty} dx \, \phi(x) \, e^{-kx} \\ P(k) &= \int_{-\infty}^{+\infty} dx \, [E_1|x|/2] \, e^{-kx} \end{aligned} \tag{30}$$

The last two transforms are simply

$$F_1(k) = I\nu \sum_{i=1}^{3} F_i \int_0^{\infty} dx \, \alpha_i e^{-(\alpha_i + k)dx} = I\nu \sum_{i=1}^{3} F_i \alpha_i/(\alpha_i + k) \tag{31}$$

and

$$P(k) = (2k)^{-1} \, ln \, [(1 + k)/(1 - k)] \tag{32}$$

By taking the Laplace transform of Eq. 29, using the definitions (30), and inserting the expressions (31) and (32), one obtains

$$\begin{aligned} K(k) + G(k) &= (1 + f) \, F(k) \, P(k) + F_1(k)/(1 + f) \\ &= (1 + f) \, F(k)(2k)^{-1} \, ln \, [(1 + k)/(1 - k)] \frac{I\nu}{1 + f} \sum_{i=1}^{3} \\ &= F_i \alpha_i/(\alpha_i + k) \end{aligned} \tag{33}$$

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