The two quantities necessary to describe the flux of fast neutrons in the active material of thickness "a" = σd are the transport cross section per cm^3, σ, and the net number of neutrons emerging per collision $1 + f = (\nu\sigma_f + \sigma_s)/\sigma$. Here σ, σ_f and σ_s are the one-group fast neutron total transport, fission, and scattering cross sections cm^3, respectively.

The actual flux at x, generated by last collisions at x′ is given by

$$m(x)\,\nu = (1 + f) \int_0^a dx' \,[E_1|x - x'|/2][m(x')\,\nu + \phi(x')/(1 + f)] \tag{23}$$

where $E_1(x)$ is the exponential integral, $\int_1^\infty dy\, e^{-xy}/y$. Let us define a function n(x) for both positive and negative values of x as follows:

$$n(x) = \begin{cases} m(x)\,\nu + \phi(x)/(1 + f) & x \geq 0 \\ m(x)\,\nu & x < 0 \end{cases} \tag{24}$$

From Eq. 23 one then obtains an inhomogeneous integral equation for n,

$$n(x) = (1 + f) \int_0^a dx' \,[E_1|x - x'|/2]\, n(x') + \phi(x)/(1 + f) \tag{25}$$

where $\phi(x)$ is zero for $x < 0$ and is given by Eq. 22 for $x \geq 0$.

The water acts as an absorber of fast neutrons, none being returned to the active material without having been moderated. One may interpret the quantity n(x) dx in the region $x < 0$ given by Eq. 25 as the absorption rate per unit area between x and x + dx of a medium having zero elastic scattering cross section for fast neutrons and an absorption mean free path just equal to the transport mean free path of fast neutrons in the active material to the right of $x = 0$. Thus the current density crossing the surface at $x = 0$ to the left is simply

$$I = \int_{-\infty}^{0} dx\, n(x) \tag{26}$$

Now one may distort Eq. 25 by letting the upper limit of the integral approach ∞ and at the same time impose the restriction that $m(x)\,\nu$ and thus n(x) (since ϕ becomes negligibly small) vanishes at $x = a + x_0$. Here x_0 is the usual extrapolated end point and a is the critical slab thickness.

$$x_0 = 0.71/(1 + f) \quad \text{and} \quad a = \sigma d \tag{27}$$

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