$$\sigma_i(g,\alpha) = Z_3 \frac{\partial \ln Z_i}{\partial \alpha_u} \Bigg|_{g_u,\Lambda \text{ fixed.}} \qquad \text{A.5}$$

$$\varepsilon(g,\alpha) = Z_3 \frac{\partial g}{\partial \alpha_u} \Bigg|_{g_u,\Lambda \text{ fixed.}} \qquad \text{A.6}$$

This equation expresses the fact that a change in gauge parameter for gauge invariant Green's functions can be reabsorbed by a change in coupling constant and in the scale of the operators. It can be used to eliminate the $\frac{\partial}{\partial \alpha}$ in Equation A.1.

Finally let us note that to lowest order $\beta(g,\alpha)$ is independent of α. This is essentially because, to lowest order, the coupling constant is unique. In other words if we change α we might change g to g', however g' (being the value of the three-point function at some point) can be expressed as a power series in g:

$$g' = g + O(g^3). \qquad \text{A.7}$$

The term of order g^3 might depend on α, however to lowest order g' and g must be equal. Therefore:

$$\beta'(g') = \mu \frac{d}{d\mu} g' \Bigg|_{g_u,\alpha_u,\Lambda \text{ fixed}} = \beta(g) + O(g^5) \qquad \text{A.8}$$

Thus that the lowest order term, of order g^3, in $\beta(g)$ must be independent of α.

Similarly the lowest order term, of order g^2, in the anomalous dimension of a gauge invariant operator (say those discussed in V) must be α-independent. This is, of course, not true of the anomalous dimensions of the vector meson and fermion fields.