a) The adjoint representation of SU(N):

The scalars are Hermitean traceless N x N matrices M of fields. The quartic self couplings are given by:

$$L = -\frac{\lambda}{8} \quad \mathrm{Tr}(M^2) \quad - \eta \ \mathrm{Tr}\ M^4 \tag{6.20}$$

b) The symmetric tensor representation of SU(N):

The scalars are represented by an N x N symmetric matrix M of (complex) fields. Under the group transformation $M \to UMU^T$. The quartic scalar interactions are:

$$L = -\lambda(\mathrm{Tr}MM^\dagger)^2 \quad - \quad \eta \ \mathrm{Tr}(MM^\dagger)^2 \tag{6.21}$$

c) The $(N,\overline{N})$ representation of SU(N):

The scalars are represented by an N x N matrix M of complex fields. The quartic self couplings are:

$$L = -\frac{\lambda}{2}\ (\mathrm{Tr}MM^\dagger)^2 - \frac{\eta}{2}\ \mathrm{Tr}\ (MM^\dagger)^2. \tag{6.22}$$

The results are shown in Table I. The main result is that all these theories are asymptotically free for N large enough. The UV stability is, as before, delicate. One must make b_o small, and additional symmetrically coupled scalars destroy the asymptotic freedom.

Our experience in these and other cases suggests that it is the size of a representation, rather than its detailed nature, that determines whether the model will be asymptotically free or not. Indeed in all the cases enumerated in Table I the renormalization group equations are identical as $N \to \infty$. If one considers larger representations the number of independent coupling constants rapidly increases and the renormalization group equations become quite complicated. For example we have analyzed a theory