$$\frac{1}{g^2}\frac{d\alpha}{dt} = A^{\alpha}_{\alpha\alpha}\,\alpha^2 + A^{\alpha}_{\alpha\beta}\,\alpha\beta + A^{\alpha}_{\beta\beta}\beta^2 + B_{\alpha}\alpha + C_{\alpha}$$

$$\frac{1}{g^2}\frac{d\beta}{dt} = A^{\beta}_{\alpha\alpha}\,\alpha^2 + A^{\beta}_{\alpha\beta}\,\alpha\beta + A^{\beta}_{\beta\beta}\,\beta^2 + B_{\beta}\,\beta + C_{\beta} \tag{6.17}$$

The fixed points of these coupled equations are easy to analyze. For each value of α there will be two roots of $\frac{d\beta}{dt} = 0$. If these roots are real then the smaller, $\beta_-(\alpha)$, is attractive whereas the larger, $\beta_+(\alpha)$ is repulsive. Similarly we define $\alpha_-(\beta)$ and $\alpha_+(\beta)$. The coupling constant plane is the pictured in Figure 12, in the case where all roots are real. We are clearly driven to the fixed point α_f, β_f satisfying:

$$\begin{aligned} \alpha_- \quad (\beta_f) &= \alpha_f \\ \beta_- \quad (\alpha_f) &= \beta_f \end{aligned} \tag{6.18}$$

as long as the initial values are small enough:

$$\begin{aligned} \alpha(o) &< \alpha_+ \qquad \beta(o) \\ \beta(o) &< \beta_+ \qquad \alpha(o) \end{aligned} \tag{6.19}$$

In practice the simplest way to search for a fixed point is to solve Equation 6.17 by iteration. We have mapped out some coupling constant trajectories on a computer, a sample result is given in Figure 13.

We shall now briefly review the results for the above mentioned theories. The same Feynman graphs as in the case of the vector representation, Figure 12, must be evaluated, except that there are now two types of scalar couplings. In each case $\alpha = \frac{\lambda}{g^2}$ and $\beta = \frac{\eta}{g^2}$.