$$T(R) < \frac{11}{4} C_2(G) \tag{4.7}$$

the theory will be asymptotically free. This requirement is not very restrictive as to the number of fermions allowed in the theory. For example if the gauge group is SU(3) $\left[C_2 = 3 \right]$ one can accommodate as many as sixteen triplets or two octets of fermions within losing asymptotic freedom.

It is therefore possible to construct a large class of theories in which the renormalization group equations for the effective coupling constant $\bar{g}(t,g)$ take the form:

$$\frac{d}{dt} \bar{g}^2 = 2\bar{g}\beta(\bar{g}) = -b_o \bar{g}^4 + b_1 \bar{g}^6 + \ldots \tag{4.8}$$

where

$$b_o = \frac{1}{8\pi^2} \left[\frac{11}{3} C_2(G) - \frac{4}{3} T(R) \right] . \tag{4.9}$$

The structure of this equation assures us that as long as $\bar{g}(0,g)$ is small enough then for large t we will have

$$\bar{g}^2(t,g) \underset{t\to\infty}{\simeq} b_o^{-1} t^{-1} + b_1 b_o^{-3} t^{-2} \ln t + 0\left(\frac{1}{t^2}\right) . \tag{4.10}$$

This will be the true asymptotic behavior of the effective coupling constant as long as $\bar{g}\ (o,g) = g$ is in the domain of attraction of the fixed point $\bar{g} = o$[38]. The size of this domain is determined by the value of the first zero of $\beta(g)$, namely if:

$$\beta(g) = o \quad \text{for} \quad g = g_1 \tag{4.11}$$

then the domain of attraction of the origin is given by:[39]

$$o < g^2 < g_1^{\ 2} \quad . \tag{4.12}$$