The effect of the fermions on β and γ is easily calculated. The fermions contribute to Z_3 via the graph shown in Figure 6 and to Z_1 via the graph shown in Figure 7. These yield the contributions

$$Z_3^F = Z_1^F = -\frac{g^2}{8\pi^2}\frac{4}{3}T(R)\ln\Lambda \tag{4.2}$$

where T(R) is defined by:

$$\mathrm{Tr}\ (\sigma^a\sigma^b) = T(R)\delta_{ab} \tag{4.3}$$

We note the elementary identity between the value of the Casimir operator for the representation R and T(R) given by:

$$rT(R) = d(R)C_2(R) \tag{4.4}$$

where d(R) is the dimension of the representation R and r the dimension (number of generators) of the group. For example, in the case of SU(N) we have for the vector representation (N) $T((N)) = \frac{1}{2}$, $C_2((N)) = \frac{N^2-1}{2N}$ whereas for the adjoint representation $T(\mathrm{ADJ.}) = N = C_2(G)$.

The equality of Z_3^F and Z_1^F is an immediate consequence of the Ward identity, Eq. 2.10, and the fact that $\tilde{Z}_1$ and $\tilde{Z}_3$ receive no contributions from fermion loops to lowest order. The resulting contributions to $\beta(g)$ and $\gamma(g)$ are

$$\beta_F(g) = +\frac{g^3}{16\pi^2}\ \frac{4}{3}\ T(R) \tag{4.5}$$

$$\gamma_v^{(F)}(g) = +\frac{g^2}{16\pi^2}\ \frac{8}{3}\ T(R) \tag{4.6}$$

These contributions are opposite in sign to those arising from the vector meson loops. Thus the fermions tend to destabilize the origin. However there is room to spare. As long as