IV. INCORPORATION OF FERMIONS

We now consider a gauge theory which includes matter fields. Fermions are easily incorporated without introducing any new coupling constants, and without destroying the asymptotic freedom. We add fermions by adding to the Lagrangian the term:

$$L_F = \bar{\psi}\ (i\not{\partial} - M - g\sigma^a \cdot \not{B}^a)\psi \tag{4.1}$$

where the σ^a are the matrices of the representation R of the gauge group G according to which ψ transforms. The fermions may have non-vanishing, but symmetric, masses since we are not considering here chiral gauge groups.

In the presence of a fermion mass the renormalization group equations are no longer valid. Instead one can derive the Callan-Symansik equations[17,18] which contain, in addition to the renormalization group operator, an inhomogeneous term arising from mass insertions. The renormalization group parameters are unaffected by these mass terms. For large Euclidean momenta the Callan-Symansik equations reduce to the renormalization group equations. This is a consequence, to any finite order in perturbation theory, of Weinberg's theorem. More precisely consider the large λ limit of a Green's function $\Gamma^{(n)}(\lambda p_i)$ for nonexceptional Euclidean momenta. The leading power in λ of $\Gamma^{(n)}$ to any finite order in perturbation theory is denoted by $\Gamma^{(n)}_{ASY}(\lambda p_i)$ and satisfies the renormalization group equation (e.g. Eq. 2.23). We should emphasize that an important assumption is being made here, namely, that the leading power behavior of perturbation theory is identical with that of the actual solution.