III. CALCULATION OF THE RENORMALIZATION GROUP PARAMETERS

We shall now proceed to calculate the renormalization group parameters β and γ to lowest non-trivial order in perturbation theory, that is to order g^3 and g^2 respectively. To calculate these functions one must calculate the renormalization constants Z_3 and Z_1 to order g^2 and use Equations 2.18-19. These constants can only depend on the renormalization point μ via the ratio $\frac{\Lambda}{\mu}$, where Λ is the ultraviolet cutoff. It is therefore sufficient to calculate the logarithmically divergent terms (of order g^2) in Z_3 and Z_1. It then follows that

$$\beta(g,\alpha) = - g \frac{\partial}{\partial \ln\Lambda} \left[\frac{Z_3^{3/2}}{Z_1} \right] \tag{3.1}$$

$$\gamma_v(g,\alpha) = - \frac{1}{2} \frac{\partial}{\partial \ln\Lambda} \ln Z_3. \tag{3.2}$$

(We denote the anamalous dimension of the vector mesons by γ_v.) The calculations are thus greatly simplified. In particular it is not necessary to specify the regularization method employed.

The wave function renormalization constant is determined by the vector meson self energy graphs shown in Figure 2. We calculate from these graphs and 2.6 that:

$$Z_3 = 1 + \frac{g^2}{16\pi^2} \left(\frac{13}{3} - \alpha\right) C_2(G) \ln\Lambda \tag{3.3}$$

where $C_2(G)$ is the value of the quadratic Casimir operator for the adjoint representation of the gauge group G. Namely:

$$\sum_{c,d} C_{acd} C_{bcd} = C_2(G) \delta_{ab}; \tag{3.4}$$

in the case of $SU(N): C_2(SU(N)) = N$.

Similarly the charge renormalization constant is determined