Of particular interest is the large λ limit of this solution, for it determines the ultraviolet behavior of the Green's functions even in the presence of mass terms (which here must arise from symmetry breaking). This limit will in turn be controlled by the large t behavior of the effective coupling constant $\bar{g}(t,g)$. If equation (2.4) admits a solution such that

$$\lim_{t\to\infty} \bar{g}\ (t,g) = g_\infty \tag{2.28}$$

then we say that g_∞ is an ultra-violet stable fixed point. The asymptotic behavior of $\Gamma^{(n)}$ is then controlled by g_∞ according to:

$$\Gamma^{(n)}(\lambda p_1,\ldots,\lambda p_n,g,\mu) \to \Gamma^{(n)}(p_1,\ldots,p_n;g_\infty,\mu)\ \lambda^{4-n-n\gamma(g_\infty)} \exp -n \int_0^{\ell n\lambda} \{\gamma\left[\bar{g}(x,g)\right] -\gamma(g_\infty)\}dx \tag{2.29}$$

so that $\gamma(g_\infty)$ is the anamalous dimension of the field.

The fixed points of the renormalization group are determined by the zeroes of $\beta(g)$; i.e. $\beta(g_\infty) = 0$. However not all such zeroes are UV stable. Thus if β has a simple zero at g_∞ this will be UV stable if and only if

$$\beta(g_\infty) = 0 \quad ; \quad \frac{d}{dg}\beta(g_\infty) < 0 \tag{2.30}$$

a zero of β at which $\frac{d\beta}{dg} > 0$ is said to be an infrared (IR) stable fixed point, since $\bar{g}$ approaches such a fixed point when $\lambda\to 0\,(t\to-\infty)$.

A theory is said to be <u>asymptotically free</u> if g_∞ vanishes. In that case $\gamma(g_\infty) = 0$ and the Green's functions can be expanding for large Euclidean momenta, in a asymptotic series in $\bar{g}(t)$ (plus a modification due to the integral in 2.29). Since in all theories $\beta(0) = 0$, the origin of coupling constant space is either UV or IR stable. It has recently been proved[28]