II. THE RENORMALIZATION GROUP EQUATIONS FOR YANG-MILLS THEORIES

In this section we shall derive the renormalization group equations for a pure Yang-Mills theory. The only restriction on the form of the theory will be the requirement that the gauge group be non-abelian and semi-simple.

The classical Yang-Mills Lagrangian density is:

$$L = -\frac{1}{2} \mathrm{Tr} \{\partial_\mu B_\nu - \partial_\nu B_\mu - g \left[B_\mu B_\nu\right] \}^2 \tag{2.1}$$

where

$$B_\mu (x) = B_\mu^{\ a} (x) \cdot t_a \tag{2.2}$$

is a matrix of Hermitien vector fields (summation over repeated indices is implied). The matrices t_a generate a semi-simple Lie Group G:

$$\left[t_a, t_b\right] = i\, C_{abc} t_c \tag{2.3}$$

and are normalized according to

$$\mathrm{Tr}\ t_a t_b = \frac{1}{2} \delta ab \tag{2.4}$$

This Lagrangian is singular due to its invariance under the gauge group. Therefore a proper quantization of (2.1) necessitates the addition of a gauge fixing term to the Lagrangian, say $-\frac{1}{\alpha} \mathrm{Tr}\ (\partial^\mu B_\mu)^2$. The presence of this term then requires the appearance of Feynman-Fadeev-Popov ghosts[31]. The net result is that the effective Lagrangian, used to derive the Feynman rules is