I. INTRODUCTION

In recent years the renormalization group has played an increasingly important role in the study of the asymptotic behavior of renormalizable field theories. This approach has acquired new importance due to the recent discovery that non-abelian gauge theories are asymptotically free.[1,2,3] In this paper we shall amplify and extend the results reported in Reference 1.

The renormalization group dates from the fundamental work of Gell-Mann and Low[4], who studied the asymptotic behavior of the photon propagator in quantum electrodynamics. The remarkable discovery of Gell-Mann and Low was that the asymptotic form of the photon propagator was determined by the zeroes of a certain, calculable, function of the coupling constant and not by the actual value of the charge. The renormalization group equations were extended by Bogolubov and Shirkov to the vertex function[5] and employed to analyze the ultraviolet and infrared behavior of quantum electrodynamics and other field theories.[6,7] (For a review of this work see Reference 8).

The basic idea underlying the renormalization group equations is very simple. A renormalizable field theory contains two types of parameters--masses or coupling constants with positive dimensions of mass (i.e. due to $M\bar{\psi}\psi$ or $\lambda\phi^3$ terms in the Lagrangian) and dimensionless coupling constants (i.e. due to $\lambda\phi^4$ or $\bar{\psi}\lambda^{\mu}\psi A_{\mu}$ terms in the Lagrangian). Coupling constants with negative dimensions of mass give non-renormalizable theories. If one considers a Green's function for large and spacelike momenta (so as to exclude any Landau singularities), then one would