absolute value of the negative copper isotope effect depends on the oxygen concentration. In optimally doped samples it is small, otherwise it depends nonmonotonically on concentration. The maximal absolute value of α is around 0.4. In a simple model one cannot hope to reproduce all the details, and our goal will be only the qualitative explanation of the most spectacular features. All phonons, whatever their nature is, contribute to the attractive interaction, and therefore, in order to explain the negative isotope effect, we have to think about some different action. In this connection we may remember that in a d-wave superconductor ordinary potential nonmagnetic scatterers with large angle scattering suppress superconductivity. Let us suppose that there exist phonons which do not affect strongly the overall ionic charge. The interaction of electrons with these phonons will be short-ranged, the same as with neutral defects. If we suppose also that the characteristic frequency of such phonons is small compared to T_c, we obtain a situation similar to elastic impurity scattering. In a d-wave superconductor this scattering will suppress superconductivity, and its effect will increase with increasing amplitude of oscillations, i. e. with decreasing mass of the ions involved. This consideration is indeed justified by exact calculations. The result is

$$\alpha \approx -\ 6.326\ g_2^2 \nu \left(\frac{\omega_{02}}{2\pi T_c}\right)^2, \tag{12}$$

where ω_{02} and g_2^2 are the frequency of Cu phonons with momentum $p_0\sqrt{2}$ and their interaction constant with electrons , ν is the density of states. The effect increases in magnitude with decreasing T_c, i. e. with underdoping. For an s-type order parameter the isotope effect is always positive.

The Raman scattering experiments [17], [18] show that indeed low frequency phonons do exist. The best candidate is the mode with $\omega_0 \approx 150\ cm^{-1}$ which corresponds to vibrations of Cu atoms. For this mode the expansion parameter of the theory $(\omega_0/2\pi T_c)^2 \sim 0.24$ (we took T_c=70K).

7. Neutron peak

Another interesting consequence of the proposed theory is the explanation of the origin and the prediction of the fine structure of the peak in inelastic neutron scattering in YBCO [19]. The clear narrow peak was observed in the superconducting phase for momentum change close to $(\pi/2,\pi/2)$ and energy change close to 41 meV ([20], and references thereon), which corresponds, approximately, to $2\Delta_{max}$, as obtained from tunneling measurements [21]. The momentum change corresponds to the transition from one extended saddle point singularity to another one. In these regions the spectrum is almost one-dimensional.

At Fig.7 the energy in the normal state is presented as function of the corresponding momentum: p_x for the a-vicinity, and p_y for the b-vicinity (dashed curve). In order to obtain from here the superconducting spectrum one has to reflect the curve with respect to the chemical potential (straight horizontal line) and introduce gaps at the intersections. Then we get the two continuos curves at Fig.7.

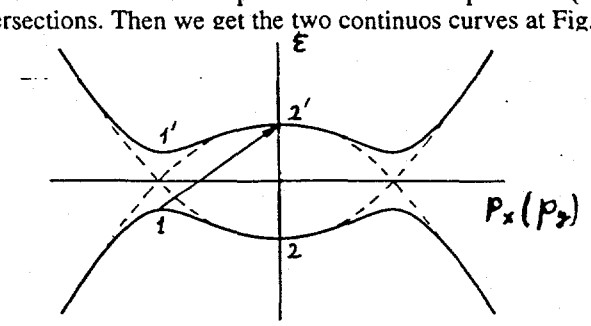

Fig. 7. Schematic presentation of the superconducting bands in the vicinity of singular points. The abscissa is p_x for the a-vicinity, and p_y for the b-vicinity. The notations 1,1',2,2' correspond to extremal points with the maximal density of states.