By evaluating $C^{(n)}(1,\bar{g})$ to the next order in perturbation theory one obtains corrections to the parton model relations. A simple calculation involving the graphs of Fig. 5 (in fact only 5a gives a non-vanishing contribution) leads to the (quark operator) result

$$\frac{C_L^{(n)}(quark)}{C_2^{(n)}(quark)} = \frac{F_L^{(n)}(q^2)}{F_2^{(n)}(q^2)} = \frac{\bar{g}^2}{16\pi^2} \cdot C_2(R) \cdot \frac{4}{n+3} , \qquad (32)$$

where $C_2(R)$ is the quadratic Casimir operator for the representation of the quarks. For the colored quark model we have $C_2(R)=4/3$. It should be emphasized again that Eq. (32) refers to the SU(3) non-singlet combinations of structure functions, e.g., the proton-neutron difference. The left hand side of Eq. (32) is an experimentally defined quantity and provides a direct determination of the effective coupling constant as a function of q^2. The smallness of $\bar{g}$ is required for self consistency of our expansions, in the large q^2 region that we are considering. One can now invert Eq. (32), at fixed q^2, to obtain

$$F_L(\omega,q^2) = 4C_2(R)\frac{\bar{g}^2}{16\pi^2}\frac{1}{\omega^2}\int_1^{\omega} d\omega'\,\omega'\,F_2(\omega',q^2), \qquad (33)$$

where we have switched to $\omega = x^{-1}$. In this way we see that

$$\frac{F_L(\omega,q^2)}{F_2(\omega,q^2)} \longrightarrow K_1\frac{\bar{g}^2}{16\pi^2} , \qquad \omega\to\infty , \qquad (34)$$

$$\frac{F_L(\omega,q^2)}{F_2(\omega,q^2)} \longrightarrow K_2\,(\omega-1)\frac{\bar{g}^2}{16\pi^2} , \qquad \omega\to 1 ,$$