IV. CORRECTIONS TO THE CALLAN-GROSS RELATION

In our discussion of the differential cross sections we have ignored possible contributions from the longitudinal structure function $F_L = F_2 - 2xF_1$. For electroproduction the ratio F_L/F_2 is known to be small already in the SLAC-MIT region. Moreover, both for electroproduction and for the neutrino reactions, asymptotically free field theories and the quark-parton model both agree that this ratio must go to zero as $q^2 \to \infty$: this is the Callan-Gross relation.[9] However, although the effects arising from F_L may indeed be small, it is nevertheless interesting to try to detect its contributions experimentally. Owing to the absence of the photon propagator this may be easier to do at large q^2 in the neutrino reaction than in electroproduction. In this section we shall consider the large q^2 properties of the ratio F_L/F_2 in the context of asymptotic freedom. This will also provide an opportunity to briefly review some of the ideas of asymptotic freedom.[17]

Let us first recall how parton model relations among structure functions are partially recovered in an asymptotically free theory. We adhere closely to the notations of Ref. (5); and for simplicity we restrict ourselves at first to SU(3) non-singlet structure functions. The analysis presented in Ref. (5), which is based on the work[18] of Wilson, Callan and Symanzik, leads to relations of the form:

$$\int_0^1 dx\, x^n F(x, q^2) = C^{(n)}\left(\frac{q^2}{\mu^2}, g\right) M_n, \qquad (28)$$