The inequality of Eq. (14) holds for all positive values of γ , α , and β , subject to Eq. (13) and to $\alpha + \beta < n_0$. We therefore seek to minimize the quantity

$$P = \gamma - f(\alpha) \tag{15}$$

within these constraints. From familiar inequalities on moments of a positive function one has that

$$\frac{\partial f(\alpha)}{\partial \alpha} > 0 , \qquad \frac{\partial^2 f(\alpha)}{\partial \alpha^2} > 0 . \tag{16}$$

Thus, for fixed α one minimizes γ within the constraint of Eq. (13) by letting β approach zero. Then from Eq. (13) it follows that

$$\gamma > \frac{\partial f(\alpha)}{\partial \alpha} . \tag{17}$$

It remains therefore to minimize

$$P(\alpha) = \frac{\partial f(\alpha)}{\partial \alpha} - f(\alpha) \tag{18}$$

with respect to α in the range $0 < \alpha < n_0$. Since in this section, conservatively, we allow for the possibility that n_o may be small, we shall simply set $\alpha = 0$. Recalling that f(0)=0, we therefore have the bound