To sufficient accuracy for our present purposes, we note the inequalities

$$F_2 \geqslant 2x F_1 \geqslant x|F_3|. \tag{5}$$

(i) Upper Bound: -

Using the inequalities of Eq. (5), together with the inequality $1 - y + y^2/2 < 1$ for $0 < y < 1$, we see that the total cross section (for the ν or $\bar{\nu}$ reactions) is bounded according to

$$\sigma < \frac{G^2}{2\pi} \int_0^{2mE} dq^2 \int_{q^2/2mE}^{1} \frac{dx}{x} F_2(x, q^2). \tag{6}$$

Now introduce a positive parameter γ, in the range $0 < \gamma < n_o$, and observe that

$$\int_{q^2/2mE}^{1} \frac{dx}{x} F_2 < \left(\frac{2mE}{q^2}\right)^{1-\gamma} \int_0^1 \frac{dx}{x^\gamma} F_2, \tag{7}$$

an inequality that follows from the positivity of F_2. According to Eq. (2), for <u>all</u> $q^2 > 0$ and all $n > -n_o$ we have the bound

$$\int_1^\infty d\omega\, \omega^{-n-2} F_2 = \int_0^1 dx\, x^n F_2 < B(n)\left[\log\left(\frac{q^2+\bar{q}^2}{\mu^2}\right)\right]^{-a_n}, \tag{8}$$