the same fractional excess of neutrons over protons, but one has twice as many particles. The larger nucleus will have twice the E_2 if we associate with each extra or unpaired particle a certain fixed energy. It follows then that E_2 should be proportional to A, or:

$$E_2 = a_3 A \left(\frac{Z}{A} - \frac{1}{2}\right)^2 = a_3 \cdot \frac{\left(\frac{A}{2} - Z\right)^2}{A} \tag{8-4}$$

The effect of Coulomb forces between protons can be taken into account by finding the energy E_3 associated with the Z protons being distributed in a sphere of radius R. This is a simple problem in electrostatics if we consider the charge uniformly distributed throughout the sphere. In this case, the potential energy of a charge Ze uniformly distributed throughout a sphere of radius R is just (3/5) $(Ze)^2/R$ in ergs (e in esu). Substituting for R from equation (8-1) and for "e" and then converting ergs to mass units, the Coulomb energy becomes:

$$E_3 = 0.000627\ Z^2/A^{1/3} \tag{8-5}$$

Our last consideration concerns the dependence of the binding energy on the even or odd numbers of protons and neutrons. It has been found empirically that there are few stable nuclei with even atomic weight A and odd atomic number Z. In fact, it can be said that the most stable nuclei tend to have both Z and A-Z even. Slightly less stability occurs in the cases Z odd, A-Z even, and Z even, A-Z odd. Clearly forces between nuclear constituents must, therefore, show a dependence on whether an even or odd number of neutrons and protons are about and so must the binding energy, An explanation has been advanced based on the idea that constituents tend to fill the nucleus' lowest energy levels and that strong forces exist between the pairs of neutrons or protons that can fill the same level. It has been empirically determined that $E_4 = \delta$ can be assigned as a correction term to our expression for the binding energy on the following basis (δ in mass units):

$$\begin{aligned} &\delta = 0 \text{ for A odd} \\ &\delta = -0.036/A^{3/4} \text{ for A even, Z even} \\ &\delta = +0.036/A^{3/4} \text{ for A even, Z odd} \end{aligned} \tag{8-6}$$

Combining the various terms, equations (8-3) to (8-6) and substituting in equation (8-1), the nuclear mass (in mass units):

$$M = 1.00893\ A - 0.00081\ Z - a_1 A + a_2 A^{2/3} + a_3 \frac{\left(\frac{A}{2} - Z\right)^2}{A} + 0.000627 \frac{Z^2}{A^{1/3}} + \delta$$

We must now evaluate the coefficients a_1, a_2, and a_3. First, a_3 is evaluated by setting dM/dZ = 0. The resulting equation between Z and A,

$$Z_A = \frac{0.00081 + a_3}{2a_3 + 0.001254A^{2/3}} \cdot A \tag{8-7}$$

is one for which M is a minimum and, therefore, gives the stablest values of Z for any A. Fitting this equation to the known stable isotopes gives a best value for a_3 of 0.083 in mass units. The other constants, a_1 and a_2, are determined by fitting the equation for M to the known data for nuclear masses, with the resulting values $a_1 = 0.00504$ and $a_2 = 0.014$. Hence the complete expression for the nuclear mass as a function of A and Z is:

$$M(A,Z) = 0.99389\ A - 0.00081Z + 0.014A^{2/3} + 0.083 \frac{\left(\frac{A}{2} - Z\right)^2}{A} + 0.000627 \frac{Z^2}{A^{1/3}} + \delta \tag{8-8}$$

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