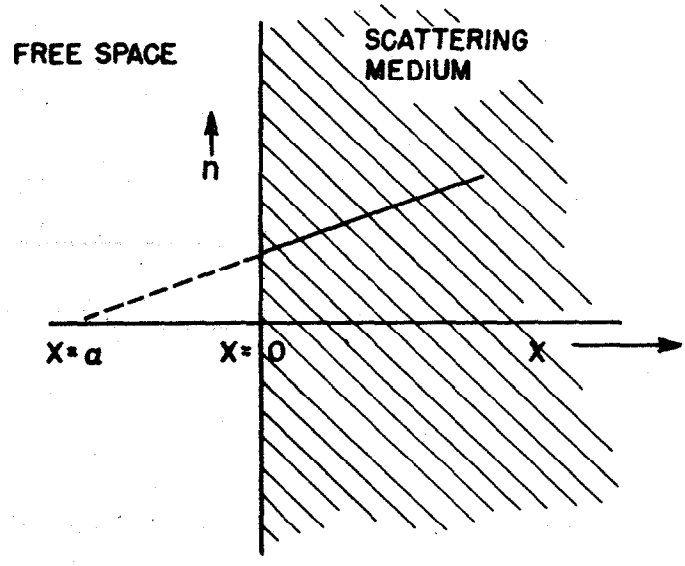

Figure 41. Neutron density behavior at boundary surface.

7.2 BOUNDARY CONDITIONS FOR THE SLOW NEUTRON DIFFERENTIAL EQUATION

In order to solve the slow neutron differential equation, the behavior of n or some function of n at the spatial boundaries must be known. Consider a finite convex (i.e., not re-entrant) medium with a neutron source in it and free space everywhere around it. What can be said of q or n at the bounding surface? To a first approximation, q or n can be taken equal to zero. This is made somewhat plausible by the argument that free space acts as a perfect sink; namely, it absorbs all neutrons and returns none. It, therefore, acts as such a heavy drain on the neutron density at the boundary that no density can be maintained there.

Actually it can be shown that a more proper boundary condition is that n or q vanish at a surface $(2/3)\lambda_t$ away from the bounding surface,* where λ_t is the neutron transport mean free path in the medium. Consider a plane bounding surface, Figure 41. The neutron density in the neighborhood of the boundary can be approximated by a linear function of the distance: $n = p(\alpha + x)$. This can be shown to satisfy equation (7-4) for a one-dimensional situation. The flux at the bounding surface is just $D\nabla n$ [see equation (6-20)] in the negative x direction. Since the gradient of n is just dn/dx, the flux is $D(dn/dx) = D \cdot p = (\lambda_t v/3)p$. The flux can be calculated by another means. In Figure 42, it is apparent that the probability that a neutron coming from the unit volume ΔV will reach the surface at point P is $\exp(-\frac{x/\cos\theta}{\lambda_t})$. Moreover, the fraction of the total solid angle included between θ and $\theta + d\theta$ is $\sin\theta\, d\theta/2$ [see equation following equation (6-5)]. Since the volume ΔV is the source of $n\Delta V/(\lambda_t/v)$ neutrons per unit time (an equal number of neutrons return to the volume per unit time in the steady state), then the total number of neutrons crossing the boundary per unit time and coming from the volume ΔV is:

$$\int_{\theta=0}^{\pi/2} \exp(-\frac{x/\cos\theta}{\lambda_t})(\sin\theta\, d\theta/2)(nv\Delta V/\lambda_t)$$

*A more refined derivation gives $0.71\,\lambda_t$.

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