CHAPTER VII

THE DISTRIBUTION OF SLOW NEUTRONS IN A MEDIUM

7.1 THE DIFFERENTIAL EQUATION FOR SLOW NEUTRONS

Calculation of the distribution of neutrons of various energies in a medium involves two distinct problems. First, there is the slowing down problem dealt with in the last chapter. The neutrons do not continue to be slowed down indefinitely, for the nuclei they collide with are not at rest but have vibrational energies corresponding to their temperature. Eventually the neutrons come into thermal equilibrium with these nuclei and show a Maxwellian distribution* corresponding to the temperature of the medium. Clearly the problem of the spatial distribution of these slowed down or thermal neutrons is quite distinct from that of the distributions of neutrons being slowed down and must be handled by different methods.

In approaching this second problem, we ask, "Given a source of thermal neutrons, what can be said about their stationary state distribution in a medium?" We seek a differential equation as our description. Let $n(x,y,z)$ be the density of thermal neutrons at x,y,z. As in the derivation of the age equation, we consider a unit volume. There are three mechanisms by which the number of neutrons in this volume element change with time: (1) diffusion into or out of the volume element, (2) absorption † or capture of neutrons in the volume element, and (3) generation of thermal neutrons in the volume element by the slowing down of fast neutrons to thermal energies. The first mechanism will yield $D\nabla^2 n$ neutrons per unit time per unit volume, as in deriving equation (6-21) of Chapter VI. The second mechanism decreases the neutron density per unit time by $-n/\theta$, where θ is the mean time for absorption or capture. The third is just the slowing down density q evaluated for thermal energies, since $q(\epsilon)$ is the number of neutrons per unit volume per unit time arriving at a particular logarithmic energy ϵ. To emphasize that q is to be evaluated for thermal energies, we write q_T. Adding the three contributions together gives the differential equation for the time rate of change of the neutron density:

$$D\nabla^2 n - (n/\theta) + q_T = \partial n/\partial t \tag{7-1}$$

where $n(x,y,z,t)dxdydz$ is the number of thermal neutrons in the volume element $dxdydz$ at time t, $(n/\theta)dxdydz$ is the number of thermal neutrons absorbed per second in the volume element $dxdydz$ at time t, and $q_T\,dxdydz$ is the number of thermal neutrons created (by slowing down) per unit time in $dxdydz$. It should be remembered that q_T is a function of x,y,z which can be determined from the age equation with the proper boundary conditions. Since $D = \lambda_t v/3$, equation (7-1) can be rewritten for the steady state:

$$\nabla^2 n - \frac{3}{\lambda_t v\theta} n + \frac{3q_T}{\lambda_t v} = 0$$

By defining:

$$\Lambda = v\theta = \text{"capture mean free path"}$$

$$L = \sqrt{\Lambda\lambda_t/3}\ \text{"diffusion length"} \tag{7-2}$$

*Number of neutrons with velocities between v and v + dv is proportional to $v^2 \exp(-mv^2/2kT)\,dv$.

†This mechanism should perhaps also have been considered in the slowing down process. But whereas orders of magnitude are such that in the slowing down the consideration of absorption is usually a refinement, here it is a necessity.

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