$$\tau = \frac{1}{3\xi(1-\cos\phi)_{av}} \int_{\varepsilon}^{0} \lambda^2(\varepsilon')d\varepsilon' \tag{6-26}$$

It is apparent that τ has the dimensions of length^2. Differentiating q with respect to ε:

$$\frac{\partial q}{\partial \varepsilon} = \frac{\partial q}{\partial \tau}\frac{\partial \tau}{\partial \varepsilon} = -\frac{\partial q}{\partial \tau}\left[\frac{\lambda^2(\varepsilon)}{3\xi\ 1-(\cos\phi)_{av}}\right]$$

Substitute into equation (6-25) and cancel out the common factor. The result is the "Age Equation":

$$\nabla^2 q = \frac{\partial q}{\partial \tau} \tag{6-27}$$

with q and τ defined by equations (6-24) and (6-26).

A considerable advantage is secured by these transformations in that equation (6-27) is identical in form to the heat conduction equation:

$$\nabla^2 T = (c\rho/k)\frac{\partial T}{\partial t}$$

Thus q is analogous to temperature and τ to time. Just as temperature will decrease with increasing time (no heat sources), so will the neutron "born" with energy* ε_0 lose its energy ε with increasing "age" τ. This can be seen by inspection of equation (6-26), where $\tau \to 0$ as $\varepsilon \to \varepsilon_0$ and τ increases with decrease in ε (or τ increases as the "time since birth" increases).

Let us apply the age equation to the point source problem. Given a point source of neutrons with energy E_0 in an infinite medium, what is the density of the neutrons as a function of E and position? First solve the age equation for q, borrowing the corresponding solution of the heat equation:

$$q = \frac{Q}{(4\pi\tau)^{3/2}}\ e^{-r^2/4\tau} \tag{6-28}$$

Identification of Q as the source strength, that is, the number of neutrons with energy E_0 introduced into the system in unit time, follows by integrating q over all space using spherical polar coordinates:

$$\int_0^\infty q \cdot 4\pi r^2 dr = 4\pi[Q/(4\pi\tau)^{3/2}]\int_0^\infty e^{-r^2/4\tau} r^2 dr = Q$$

since

$$\int_0^\infty \exp(-\alpha x^2)x^2 dx = \sqrt{\pi}/(4\alpha^{3/2}).$$

Let us examine the form of the solutions equation (6-28). In Figure 39, a sketch of q versus r for large and small values of τ shows that as E decreases from E_0 (i.e., as τ increases), the space

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* Although ε is conveniently referred to as the neutron's energy, it should be remembered that ε is the natural logarithm of the neutron energy.

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