

and each collision being followed by one mean free path (λ) movement. As shown in equation (6-18), λ_t and λ are related by the equation:

$$\lambda_t = \frac{\lambda}{1 - (\cos \phi)_{av}} \tag{6-19}$$

For isotropic scattering, equation (6-4), since $(\cos \phi)_{av} = 2/(3A)$, the relation between the transport mean free path and the mean free path is:

$$\lambda_t = \frac{\lambda}{1 - \frac{2}{3A}}$$

When $A \gg 1$, $\lambda_t \approx (1 + \frac{2}{3A}) \lambda \approx \lambda$. As in kinetic theory, the neutron current density S is related to the diffusion coefficient by:

$$S = -D \nabla n \text{ (neutrons/cm}^2\text{/sec)} \tag{6-20}$$

This is a vector equation, with S in the direction of the gradient. Notation is standard:

$\nabla n = i_x(\partial n/\partial x) + i_y(\partial n/\partial y) + i_z(\partial n/\partial z)$ with i_x , i_y , and i_z the unit vectors in the x , y , and z directions. Consider the face $dydz$ of the volume element at (x,y,z) . The neutron current out of this face (in the negative x -direction) is $D(\partial n/\partial x)$ so that the number of neutrons per unit time in the energy range between ϵ and $\epsilon + d\epsilon$ going out of the face $dydz$ at (x,y,z) is $D(\partial n/\partial x) dydz d\epsilon$. At the opposite face at $(x + dx, y, z)$ the number of neutrons per unit time in the energy range $d\epsilon$ coming into the volume element (negative x -direction) is:

$$D \left(\frac{\partial n}{\partial x} + \frac{\partial^2 n}{\partial x^2} dx \right) dydz d\epsilon$$

The net gain of neutrons per unit time in the volume element is thus $D(\partial^2 n/\partial x^2) dx dydz d\epsilon$. Taking other pairs of faces and adding up, we obtain:

$$\begin{aligned} \text{Neutrons in energy range between} \\ \epsilon \text{ and } \epsilon + d\epsilon \text{ diffusing into the} \\ \text{volume } dx dy dz \text{ per unit time} \end{aligned} = D \nabla^2 n dx dy dz d\epsilon \tag{6-21}$$

with ∇^2 the Laplacian $= (\partial^2/\partial x^2) + (\partial^2/\partial y^2) + (\partial^2/\partial z^2)$.

Now consider the other source of neutrons, those in the volume element which are slowed down to the proper energy range (between ϵ and $\epsilon + d\epsilon$). The number of collisions of a neutron in unit time is v/λ . If this is multiplied by the average change in ϵ per collision, that is by ξ , the result $\xi v/\lambda$ is the loss of ϵ per unit time. Representing values of ϵ as points on a straight line, Figure 38, a neutron can be visualized as moving down the ϵ line with a velocity $\xi v/\lambda$

$(\epsilon = \text{LOG}_e E)$

NEUTRON PROCEEDS DOWN ϵ AXIS WITH SPEED = $\xi v/\lambda$

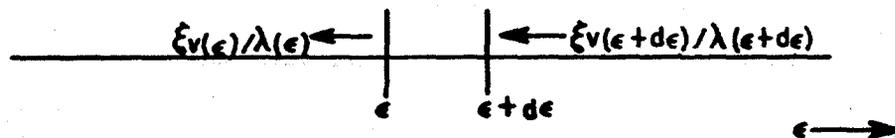


Figure 38. Slowing down of neutrons: appearance on ϵ axis.

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