successive terms may be replaced by λ_1 itself. This is valid if the λ's change very little between successive collisions. In this manner, the coefficient of λ_1 is approximated by:

$$2[\lambda_1 + \lambda_1 C + \lambda_1 C^2 + \ldots] = \frac{2\lambda_1}{1-C}; \quad C = \frac{2}{3A}$$

Consistent with our approximation, the finite series has been considered to be infinite. It can be readily shown that the error becomes negligible for $A >> 1$ (and absorption processes are very few compared to scattering). Substitution of this approximation into (6-13) yields

$$(r^2)_{av} = \frac{2\lambda_1^2}{1-C} + \frac{2\lambda_2^2}{1-C} + \frac{2\lambda_3^2}{1-C} + \ldots \frac{2\lambda_n^2}{1-C} = \frac{2}{1-C} \sum_{i=1}^{n} \lambda_i^2$$

where $C = 2/(3A)$ as before. The sum may be written as an integral if the change of energy is small for each collision:

Number of collisions between $E + \Delta E$ and $E = \dfrac{\log_e(E+\Delta E) - \log_e E}{\xi}$

$$= \Delta E/(\xi E)$$

Thus

$$r^2_{av} = \frac{2}{1 - \frac{2}{3A}} \sum_{i=1}^{n} \lambda_i^2 = \frac{2}{\xi(1 - \frac{2}{3A})} \int_E^{E_o} \frac{\lambda^2(E')dE'}{E'} \tag{6-14}$$

The integration variable is primed to distinguish it from the limits.

In (6-14) ξ is the average decrease in the natural logarithm of the energy per collision, defined in equation (6-7). The dependence of the mean free path on the energy has been noted by writing $\lambda^2(E)$. E_o is the initial neutron energy at the source, and E is the energy of the neutron at the particular position $\vec{r}$. Equation (6-14) the average displacement-squared for neutrons of energy E when the scattering is done by heavy nuclei.

It is often convenient to use logarithmic variables, in which case the average of the displacement-squared is:

$$r^2_{av} = \frac{2}{\xi(1-\frac{2}{3A})} \int_{\log_e E}^{\log_e E_o} \lambda^2(\epsilon')\, d\epsilon' \quad \text{with } \epsilon' = \log_e E' \quad (A>>1) \tag{6-15}$$

As was mentioned previously, the formula for $(r^2)_{av}$ can be derived exactly. The result [Ricerca Sci. 7:13(1936)] is:

$$\begin{aligned} r^2_{av} = {} & 2\lambda^2(0) + 2\lambda^2(a) + 2\int_0^a \lambda^2(x)dx + 2\lambda(0)\int_0^a \lambda(x)e^{-x/2}\,dx \\ & + 2\lambda(0)\lambda(a)e^{-a/2} + 2\lambda(a)\int_0^a \lambda(x)e^{-(a-x)/2}\,dx \\ & + 2\int_0^a \lambda(u)du \int_0^{a-u} \lambda(u+x)e^{-x/2}dx \end{aligned} \tag{6-16}$$

where $x = \log_e(E_o/E')$; $\quad a = \log_e(E_o/E)$

a-93-6