Now, in the above, the vector $\vec{l}_2$ was kept fixed and $\vec{l}_4$ averaged. If $\vec{l}_4$ is kept fixed and $\vec{l}_3$ allowed to rotate about $\vec{l}_2$ we find, in a manner similar to that used in deriving (6-9), that:

$$[\cos(1,3)]_{av} = [\cos(1,2)]_{av} \cdot \cos(2,3) \tag{6-10}$$

However, since vectors $\vec{l}_1$ and $\vec{l}_2$ are successive vectors the average of the cos (1,2) is a constant; that is, when $\vec{l}_2$ rotates about $\vec{l}_1$, the angle (1,2) remains constant. Substitution of equation (6-10) into equation (6-9) proves the theorem stated in equation (6-8). The theorem may be generalized to apply to any succession of such vectors averaged over angle ψ (as shown in Figure 36):

$$[\cos(a,z)]_{\psi av} = \cos(a,b) \cdot \cos(b,c) \cdot \cos(b,c) \ldots \cos(y,z) \tag{6-11}$$

$$(\vec{l}_a, \vec{l}_b, \vec{l}_c, \ldots \vec{l}_z \text{ are successive vectors})$$

Now consider the average over l. If λ_n is the mean free path, then the probability that l_n lies between l_n and $l_n + dl_n$ is $\exp(-l_n/\lambda_n)$. The average of l_n and l_n^2 can be readily determined:

$$(l)_{av} = \left[\int_0^\infty l \exp(-l/\lambda)\, dl\right] / \left[\int_0^\infty \exp(-l/\lambda) dl\right] = \lambda \tag{6-12}$$

$$(l^2)_{av} = \left[\int_0^\infty l^2 \exp(-l/\lambda) dl\right] / \left[\int_0^\infty \exp(-l/\lambda)\, dl\right] = 2\lambda^2$$

(The subscripts "n" have been omitted for simplification of notation.) The first relation of equation (6-12) is in a sense a definition, since the average of l is just the mean free path.

Returning to the original equation for r^2, we may now write the equation for r^2 averaged over the ψ's and l's:

$$r^2_{l,\psi av} = 2\lambda_1^2 + 2\lambda_2^2 + 2\lambda_3^2 + \ldots 2\lambda_n^2 + 2[\lambda_1\lambda_2 \cos(1,2) + \lambda_1\lambda_3 \cos(1,2)\cos(2,3) + \ldots \lambda_2\lambda_3 \cos(2,3) + \ldots] \tag{6-13}$$

In the case of hydrogen, this calculation can be completed in an exact way. Generally it is convenient to make some approximations. Let us assume that it takes a large number of collisions to produce a small change in energy. This will be valid for neutron collisions with heavy atoms ($A >> 1$). The various angles of equation (6-13), (1,2), (2,3), etc., may take on all values from 0 to π. We must average over the various possible angles, recalling that these angles are the successive angles of scattering as measured in the laboratory system, angle ϕ of Figure 29. As we have proved in equation (6-4), the average of the cosines of these angles for isotropic scatterers is $2/(3A)$, a number that decreases with increasing A. This fact may be used to advantage if we note the coefficient of λ_1 in (6-13):

$$2[\lambda_1 + \lambda_2 \cos(1,2) + \lambda_3 \cos(1,2)\cos(2,3) + \ldots]$$

Since successive terms when averaged over scattering angles, will have increasing powers of $2/(3A)$, they will diminish rapidly in the approximation being considered, so that the various λs occurring in

a-93-64