with cos (m,n) = cosine of angle between $\vec{l}_m$ and $\vec{l}_n$

This theorem may be proved by considering $\vec{l}_1$ fixed. Then, with $\vec{l}_2$ fixed, consider the vector $\vec{l}_4$ to be averaged by allowing $\vec{l}_4$ to rotate around $\vec{l}_3$. Figure 37 shows the three vectors $\vec{l}_1$, $\vec{l}_3$, and $\vec{l}_4$ arranged to form edges of a tetrahedron. The base plane has been constructed by passing a plane perpendicular to $\vec{l}_3$ and intersecting $\vec{l}_1$ at unit length. The figure is to be used in the averaging process where $\vec{l}_4$ is allowed to rotate about $\vec{l}_3$, so that the angle ψ indicated on the diagram is to be varied from 0 to 2π in the averaging process. Considering the triangles APB and BPC of Figure 37 it is apparent from elementary trigonometry that

$$AP = 1;\ AB = \sin(1,3);\ PB = \cos(1,3);\ BC = PB \cdot \tan(3,4) = \cos(1,3)\tan(3,4)$$

$$PC = PB/\cos(3,4) = \cos(1,3)/\cos(3,4)$$

The quantity $(AC)^2$ can be found by use of the cosine law in triangle APC and in triangle ABC. Equating these two values gives:

$$(AC)^2 = (AP)^2 + (PC)^2 - 2(AP)(PC)\cos(1,4) = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos\psi$$

Solving this equation for cos (1,4) and substituting the trigonometric formulas for the various sides (AP, PC, AB, BC) yield the following:

$$\cos(1,4) = \cos(1,3)\cos(3,4) + \sin(1,3)\sin(3,4)\cos\psi$$

As $\vec{l}_4$ rotates around $\vec{l}_3$ the cos ψ averages out to zero. The angle between these two vectors (3,4) being constant during this averaging process means that the average of the cos (1,4) becomes:

$$[\cos(1,4)]_{av} = [\cos(1,3)]_{av} \cdot \cos(3,4) \qquad (6\text{-}9)$$

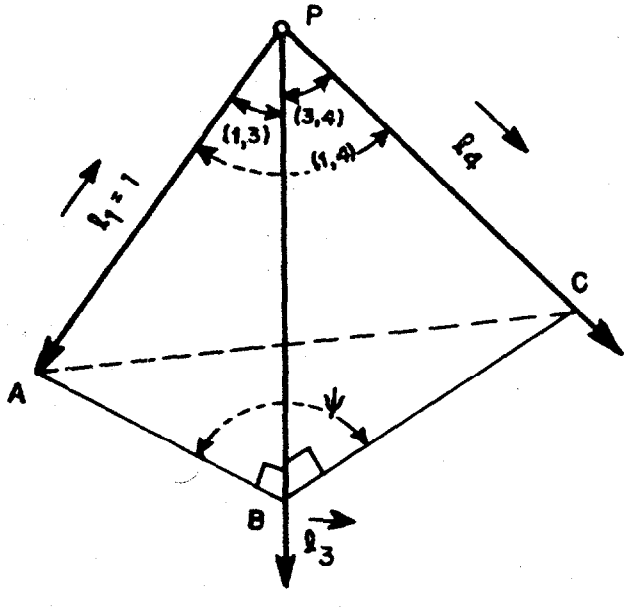

Figure 37. Averaging over azimuthal angle (ψ).

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