Let us re-examine the ratio of neutron kinetic energies after and before a collision, E'/E of equation (6-2). The maximum and minimum values are:

$$\frac{E'}{E} = \frac{A^2 + 1 + 2A\cos\theta}{(A+1)^2} = \begin{cases} 1 \text{ for } \theta = 0 \text{ (glancing collision)} \\ \left(\frac{A-1}{A+1}\right)^2 \text{ for } \theta = \pi \text{ (head on-collision)} \end{cases} \tag{6-5}$$

In collisions with hydrogen, $A = 1$, the limits are thus zero and unity. For heavier atoms, it is impossible to bring the neutron to rest. When $A >> 1$, the minimum is $(A-1)^2/(A+1)^2 = 1 - (4/A) + (8/A^2)\ldots$, so that for $A = 100$, the greatest possible loss of neutron kinetic energy after a single scattering is 4%. For $A = 200$, it is 2%.

Now what is the relative probability of a neutron's having an energy E' (between the limits just described) after a collision? Assuming isotropic scattering, the probability dp that the neutron is scattered into the solid angle between θ and $\theta + d\theta$ is equal to that solid angle divided by the total solid angle 4π:

$$dp = (\text{Solid angle between } \theta \text{ and } \theta + d\theta)/4\pi = 2\pi \sin\theta\, d\theta/4\pi = \sin\theta\, d\theta/2$$

The relation between dp and the range of final energy which corresponds to this range of angles is found by differentiation of equation (6-2).

$$dE' = -\frac{2AE\sin\theta d\theta}{(A+1)^2} = -E\frac{4A}{(A+1)^2}\,dp$$

The negative sign means that increasing θ corresponds to decreasing E. Hence

$$\begin{Bmatrix} \text{The probability that the} \\ \text{final energy is between} \\ E' \text{ and } E' + dE' \end{Bmatrix} = dp = \frac{(A+1)^2}{4A}\frac{dE'}{E} \tag{6-6}$$

where the negative sign has been dropped so dE' is considered positive. Equation (6-6) means that the *probability of the final energy being E' is independent of E'*. Figure 30 is a graph of the distribution function $p(E')$ versus E'/E, showing it to be a constant, $(A+1)^2/4A$, between $E'/E = (A-1)^2/(A+1)^2$ and $E'/E = 1$. As a check on the normalization of (6-6), it should be noted that the area under the distribution curve is unity:

$$\int p(E')\,dE' = \int_{\left(\frac{A-1}{A+1}\right)^2}^{1} [(A+1)^2/4A]\,d(E'/E)$$
$$= [(A+1)^2/4A]\left[1 - \left(\frac{A-1}{A+1}\right)^2\right] = 1$$

Using this distribution function, we could now calculate the average of E'/E. However, it is more convenient to consider the average of the natural logarithm of the energy ratio that is, $\log_e(E'/E')$. This is due to the fact that, since the per cent loss in energy is on the average the same, the neutron's energy decreases in successive collisions, as shown in Figure 31. In each collision, it is the $\log_e E$ rather than E which changes by a relatively fixed amount. Evaluation of the average of $\log_e(E/E')$ proceeds as follows:

$$\zeta = [\log_e(E/E')]_{av} = \int_{\text{Min } E'}^{\text{Max } E'} \log_e(E/E)\, p(E')\,dE'$$
$$= \int_{\frac{E(A-1)^2}{(A+1)^2}}^{E} \log_e(E/E')[(A+1)^2/4A]\,\frac{dE'}{E}$$

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