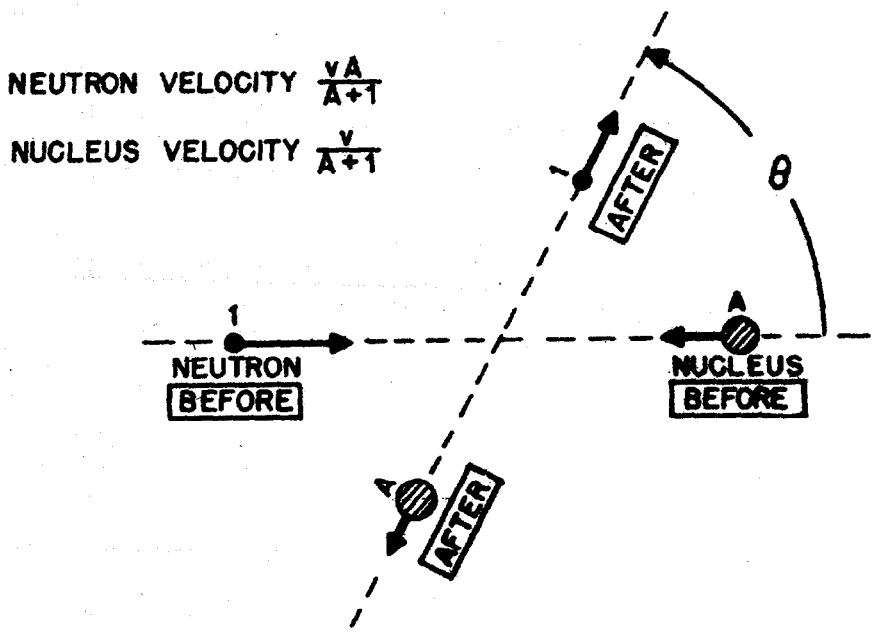

Figure 28. Neutron Scattering: Center of gravity system.

Now the transformation from the C system to the R system is readily made if we consider that the C and R systems move relative to each other with a velocity equal to the velocity of the center of mass in the laboratory system. This velocity is the same as the initial velocity of the nucleus A in the C system, or just $v/(A + 1)$. Taking this vector and adding it to the "after" velocity of the neutron in the C system $[vA/(A + 1)]$ gives us the "after" velocity (v'_α) of the neutron in the R system (see Figure 29). It is apparent that the angle of scattering in the C system, that is, θ, is not the same as the angle of scattering in the R system, ϕ. It is readily proved (by the sine law) that

$$A \sin(\theta - \phi) = \sin \phi$$

or

$$\tan \phi = \frac{A \sin \theta}{1 + A \cos \theta} \qquad (6\text{-}1)$$

Both expressions are equivalent. With either, it is possible to convert scattering angles from one system to the other.

The neutron energy before the collision in the R system is $mv^2/2$. After the collision, it is $mv'^2/2$. Thus the ratio of neutron energies after and before collision as observed in the R system is $(v'/v)^2$. This ratio can be determined from Figure 29 by the cosine law:

$$v'^2 = [v/(A + 1)]^2 + [vA/(A + 1)]^2 + [2Av^2/(A + 1)^2] \cos \theta$$

$$\frac{\text{K.E. after collision}}{\text{K.E. before collision}} = \frac{E'}{E} = \frac{A^2 + 1 + 2A \cos \theta}{(A + 1)^2} \qquad (6\text{-}2)$$

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