The amplitude of the neutron wave would be $\sqrt{\nu\sigma/4\pi}/r$. Let us estimate the amplitude of the scattered wave in the reinforced directon. This will be the amplitude for the single atom times the number of atoms participating in the scattering, that is

$$N \times N \times M \times \sqrt{\nu\sigma/4\pi}/r$$

The intensity in this direction will thus be $N^4M^2\nu\sigma/(4\pi r^2)$ neutrons/cm^2/sec.

Now the angular spread* of the beam after scattering is just λ/ beam diameter or $\lambda/(Na \sin\theta)$. Thus the area of the scattered beam in the reinforced direction is r^2 times the angular width squared or $r^2 \lambda^2/(Na\sin\theta)^2$. Multiplying this by the intensity gives the number of neutrons scattered in the reinforced direction:

$$[N^4M^2\nu\sigma/(4\pi r^2)]\ [r^2\lambda^2/(Na \sin\theta)^2]$$

$$= \nu\sigma N^2M^2(\lambda/a)^2/(4\pi \sin^2\theta) \text{ neutrons/sec}$$

It is to be noted that in the above derivation, the attenuation of the neutron intensity in passing through the M layers has been neglected. If we further simplify by considering all reflections are first order ($n=1=2a \sin\theta/\lambda$), then the total number of neutrons per second in the reinforced direction is

$$\text{Neutrons/sec scattered} = \nu\sigma N^2M^2/\pi \qquad (5\text{-}15)$$

Now if the crystal were a perfect reflector, then all incident neutrons would be removed from the beam, that is, just ν (neutrons/cm^2/sec) times $(Na\sin\theta)^2$, the beam area. This number can be considered the upper limit for the scattered beam. Thus

$$\nu\sigma N^2M^2/\pi < \nu N^2a^2 \sin^2\theta$$

or

$$\sigma M^2 < a^2(\pi/4)(\lambda/a)^2 \qquad (5\text{-}16)$$

the latter since $\sin\theta\ \lambda/2a$ for first order effects. If we assume $\lambda \simeq a$, the inequality may be written as an order of magnitude relation $M < a/\sqrt{\sigma}$.

What is the significance of this inequality? It means that the layers beyond $M_o = a/\sqrt{\sigma}$ do not contribute to the scattered beam. Put another way, the beam does not penetrate beyond M_o layers. Numerically, if a is 3×10^{-8} cm and σ about 4×10^{-24} cm^2/atom, then M_o is of the order of $3\times10^{-8}/2\times10^{-12} \simeq 10^4$. Thus about 10^4 planes play a vital part in Bragg reflection. The depth of penetration will be $M_o a$ or $\sim 5 \times 10^{-4}$ cm = 5 microns.

Throughout the foregoing discussion, it has been assumed that the only attenuation of neutrons is due to Bragg-like reflection. Consider a heterogeneous beam of neutrons incident on a large perfect crystal. Those neutrons that satisfy the Bragg condition will be scattered out of the beam in penetrating the first few microns. However, those that do not satisfy the Bragg condition generally will not be transmitted without some loss in intensity. The reasons for this are:

1. The presence of isotopes makes a random irregularity resulting in incoherent scattering for all velocities.

2. The random variation of the spin direction of nuclei also results in incoherent scattering.

3. Even if the crystal were regular, the thermal motions of the atoms would contribute to non-Bragg scattering.

4. The crystal atoms will generally have a finite absorption cross section even though $\sigma_a << \sigma_s$.

*The formula should be multiplied by a constant depending on shape of beam area. See any standard physical optics text.

a-93-5