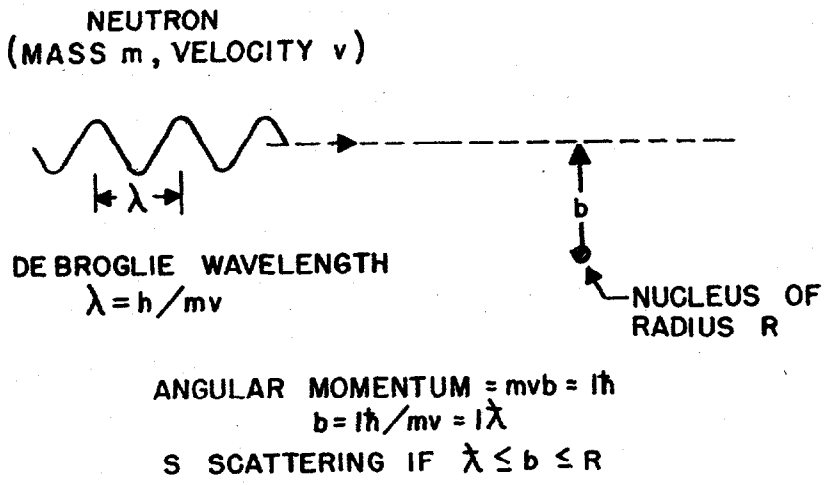

Figure 21. Quantitizing the angular momentum.

very small. Since $\delta = 2\pi(a/\lambda)$, we can replace $\sin^2\delta$ by δ^2 and obtain

$$\sigma_{scat} = (4\pi\hbar^2/m^2v^2)(4\pi^2a^2/\lambda^2) = 4\pi a^2 \ (E \to 0) \tag{5-10}$$

The simplification follows from the relation $\lambda = h/mv$.

In the case of bound states, the scattering formula (5-9) can be shown to reduce to

$$\sigma_{scat} = \frac{4\pi\hbar^2}{m[\epsilon + (E/2)]} \tag{5-11}$$

ϵ = binding energy of neutron when bound to the nucleus

E = kinetic energy of incident neutron

It is not always necessary for ϵ to be positive or "real." There are cases where "virtual" states of a nucleus may exist, these virtual states being characterized by negative ϵ. In such cases, the neutron wave function will be periodic inside and outside the nucleus but will have a larger amplitude inside than outside for that particular energy associated with the virtual state. When ϵ is negative, the absolute value is used in equation (5-11). An example of this occurs in the scattering of neutrons by protons. (The protons are considered "free" i.e., not bound chemically, in the following.) The cross section for scattering of slow neutrons by protons is dependent on what states are possible for the deuteron (combination of neutron and proton). There are two types of states, "singlet" and "triplet" states, associated with zero and unit total spin quantum number:

S = 0 (one singlet states; antiparallel spins)

S = 1 (three triplet states; parallel spins)

(Number of states = 2S + 1)

It has been observed experimentally that the stable ground state of the deuteron is the S = 1 state, so that the triplet deuteron state has a positive ϵ. The S = 0 state is probably virtual, although

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