The occurence of Ω in $\rho(E)$ shows that as the volume, in which the ejected particle is confined, is allowed to become infinitely large the density of states becomes infinite or the particle can be ejected with any energy (i.e., a continuum of possible energy states). The transition probability does not become infinite, however, for the matrix elements contain Ω in such a way that M is proportional to Ω^{-1}. Thus the probability of a transition to a state where the ejected particle or photon has a momentum p_e (subscript for ejected or outgoing particle after the collision) is proportional to $M \cdot p_e^2/v_e$. Then for a single incident particle with velocity v_i colliding with a single nucleus it follows from equation (2-4) that the probability of a collision (per unit time) is $v_i\sigma$ where σ is the cross section for incident particles of velocity v_i to result in outgoing particles of velocity v_e. Equating the probabilities and incorporating the various constants into M we obtain an equation for the cross section:

$$\sigma = M' \cdot \frac{p_e^2}{v_e} \frac{1}{v_i} \tag{2-11}$$

Equation (2-11) is too general for one to understand fully its significance. Let us apply the result to several specific situations:

Elastic Scattering

In an elastic scattering collision initial and final velocities are equal, i.e., $v_i = v_e$. Substituting this into (2-11) reduces the cross section to

$$\sigma = M' \; m^2$$

This means the cross section is proportional to M'. For slow neutrons, where the energy range is small, the elastic scattering cross section will not depend appreciably on the neutron energy. It should be pointed out that for elastic scattering from nuclei of small mass number initial and final velocities are equal only in the center of gravity system and that it is in this system that the foregoing derivation is true.

Absorption Processes

Suppose the recoil is heavy so that the ejected (small mass) particle has almost all the kinetic energy in the final state. The situation is shown in Figure 6. A neutron of mass m, velocity v_i hits the target nucleus, is absorbed and a new light (small mass) particle is ejected with mass m' and velocity v_e. Conservation of energy requires that the kinetic energy of the ejected particle must be equal to the kinetic energy of the incident neutron plus whatever energy Q is available from the nuclear reaction:

$$m' v_e^2/2 = m v_i^2/2 + Q$$

$$\text{or} \qquad p_e^2 = (m')^2 \; [(m/m')v_i^2 + (2/m')Q]$$

Noting that $p_e^2/v_e = m' p_e$ and substituting in (2-11) gives the following expression for the cross section:

$$\sigma = M'(m')^2 \frac{1}{v_i} [(m/m')v_i^2 + (2/m')Q]^{1/2}$$

When Q is positive (exothermic reaction) and v_i is small the cross section is proportional to $1/v_i$—the so-called 1/v law. A negative Q and a very small v_i does not make physical sense, giving an imaginary cross section and showing that the formula does not cover the case. σ should be zero for such a situation.

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