2.4 THE GENERAL FEATURES OF COLLISIONS *

In any collision process the "initial state" consists of a particle incident on a target nucleus and the "final state" consists of an ejected particle and the recoil nucleus. For a given energy of the initial state there are a number of possible energies of the ejected particle as well as a number of possible energies of the recoil nucleus, provided these energies are consistent with energy conservation.

If the magnitude of the momentum of the ejected particle and its direction of motion are fixed, then momentum conservation fixes the momentum of the recoil and thereby the recoil's kinetic energy. The conservation of energy law then fixes the state of excitation of the recoil nucleus. Thus specifying the momentum (magnitude and direction) of the ejected particle specifies the final state completely.

Suppose the ejected particle has an energy E. Then what is the probability that a transition between initial and final states will occur? Assuming such transitions obey the general laws of quantum mechanics, it can be shown that the probability that a transition will occur in which the energy of the ejected particle is between E and E + dE can be written as a product $M\rho$ where $M = |H|^2/\hbar^2$ and $\rho(E)$ is the density of possible final states in the neighborhood of E. H is a matrix element which will be discussed qualitatively herewith and in a later chapter; $\hbar$ is Planck's constant divided by 2π.

The density function can be derived from statistical mechanics considerations. In a number of instances the variation of this factor will be found to be more effective than the variation of M. Using a familiar technique in quantum mechanics, the ejected particle is imagined to be in large box of volume Ω. This volume will be infinite in any practical case. The number of states of the ejected particle which will have an energy E in this box is proportional to the volume in phase space corresponding to this energy. With Cartesian coordinates where h is the linear dimension of a cell in phase space (a cell can contain one state), then the number of states for which x is between x and x + dx, y between y and y + dy, etc. and p_x is between p_x and $p_x + dp_x$, p_y between p_y and $p_y + dp_y$, etc. is just $dx\,dy\,dz\,dp_x\,dp_y\,dp_z/h^3$. Integrating over configuration space (x,y,z,) reduces this to $\Omega\,dp_x\,dp_y\,dp_z/h^3$ or (Ω/h^3) times the momentum volume element. It is apparent then that the number of states for which the *total* momentum $p = \sqrt{p_x^2 + p_y^2 + p_z^2}$ is between p and p + dp is just Ω/h^3 times the volume element between p and p + dp. This volume element, a spherical shell in momentum space, is $4\pi p^2 dp$. Thus

$$\text{Number of states with momentum between } p \text{ and } p + dp = dN = (\Omega/h^3)\,4\pi p^2 dp \tag{2-9}$$

The density of states per unit energy range, $\rho(E)$, can now be calculated from (2-9) by changing from momentum to energy variables. For particles (α, p, n, etc.) $E = mv^2/2 = p^2/2m$ so that $dE = p\,dp/m = v\,dp$. For photons $E = h\nu = hc/\lambda$ or since $\lambda = h/p$ this means $E = pc$ or $dE = c\,dp$. (In both cases dE/dp is the particle or photon velocity.) Substituting these into (2-9):

Number of states per unit energy interval with energy between E and E + dE

$$\rho(E) = \frac{dN}{dE} = \frac{4\pi\Omega}{h^3}\cdot\frac{p^2}{v} \tag{2-10}$$

where $p^2/v = m^2 v$ for particles

$= h^2\nu^2/c^3$ for photons

*In all of the discussion it is assumed that the ejected particle is in its "ground state," i.e., not excited.

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