While this representation is convenient it must be understood that the nucleus is not an area in the usual geometrical sense. Among other things the area is dependent on the energy of the incident particles. For neutrons this dependence may be understood qualitatively by considering the variation of the de Broglie wavelength of neutrons with energy. The fundamental relationship between the wavelength λ of a particle and its momentum p is $\lambda = h/p$ where h is Planck's constant. In terms of neutron energy this becomes:

$$E = mv^2/2 = (mv)^2/(2m) = p^2/2m \quad \text{or} \quad p = \sqrt{2mE}$$

$$\text{Thus } \lambda = h/p = h/\sqrt{2mE} \quad \text{for } v << c \tag{2-2}$$

The condition that v must be less than c, the velocity of light, is necessary since the non-relativistic expression for the kinetic energy has been used. (This limitation is negligible since the rest mass of the neutron is 931 Mev.) Expressing the neutron energy E in electron-volts and substituting $h = 6.61 \times 10^{-27}$ erg-sec., $m = 1.675 \times 10^{-24}$ gm, 1 electron-volt = 1.601×10^{-12} erg in the equation (2-2) yields the following convenient relation:

$$\lambda = \frac{0.286}{\sqrt{E}} \times 10^{-8}\text{cm} \qquad \text{(E in electron-volts)} \tag{2-3}$$

Now, the distance between atoms in a solid is of the order of 3 or 4 Angstroms, that is $\sim 3 \times 10^{-8}$ cm. Putting numerical values into equation (2-3) shows that neutrons of about 0.01 ev have a characteristic wavelength of approximately interatomic dimensions, or very many times larger than any nuclear dimensions. To have a wavelength equal to the nuclear diameter of a medium weight nucleus, say 10^{-12} cm, the neutron energy would have to be about 10 Mev.

Now, a nucleus of diameter 10^{-12}cm, the diameter being defined as twice the range of nuclear forces, will have a "geometrical" cross section of about 10^{-24} cm^2 or 1 barn. If the neutron were a point particle then it would be reasonable to anticipate that all cross sections should be of the order of barns. However, as shown in equation (2-3) if the neutron behaves in a manner consistent with the fundamental basis of wave mechanics then a neutron only can be considered a "point" particle with respect to the nucleus when its wavelength is at least less than nuclear dimensions. This latter will be true only for fast neutrons, that is, with energies at least greater than 1 Mev. For slower neutrons the wavelength increases so that for thermal neutrons of 1/40 ev energy λ is about 2×10^{-8} cm. In this case it is the nucleus which is the "point" particle relative to the neutron so that the neutron size might be expected to determine the cross section. Thus σ should be of the order of λ^2 for slow neutrons. For thermal neutrons one might expect cross sections as large as 10^{-16} cm^2 or 10^8 times the fast neutron cross sections. Actually more rigorous theoretical analysis shows that λ^2 is an outside figure and that $\sigma < \lambda^2$.

The foregoing discussion is intended to present one argument showing that it is not possible to assign a unique geometrical cross section to a nucleus valid for all energies of incident neutrons, since in the range of energies generally considered in practical cases, say from 10^{-3} to 10^7 electron-volts, the neutron's "size" varies from 10^{-8} to 10^{-12} cm. Factors other than neutron size will be found to affect the cross section; these will be discussed in a later chapter.

In keeping with the idea that the cross section is not a strictly geometrical quantity the relation stated in the first paragraph of this section should be solved for σ:

$$\sigma = \frac{\text{Processes per nucleus per unit time}}{nv} \tag{2-4}$$

n = neutrons per unit volume in incident beam

v = neutron velocity

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