The exact threshold energy may be calculated by considering the initial momentum of the system. The total momentum of the colliding particles after the reaction will have to be the same as the total momentum before. Assuming the Li^7 to be initially stationary and the alpha particle to have a velocity v, then the total initial momentum of the system* is just 4v. The velocity of the center of gravity v_c is this momentum divided by the total mass of the system or 4v/11. The compound nucleus formed at collision (B^{11}) thus has a kinetic energy of $11v_c^2/2$, that is one half the total mass, 11 units, times the square of v_c. Substituting 4v/11 for v_c yields $(4/11) \cdot 4v^2/2$ or just four-elevenths of the kinetic energy of the incident alpha particle. The balance or seven-elevenths of the kinetic energy of the incident alpha particle is thus available for nuclear excitation. For the reaction to be just possible this fraction of the alpha's kinetic energy must be just equal to the negative reaction energy Q:

$$-Q = (7/11)\,(\text{Threshold K.E. of alphas}) \tag{1-2}$$

$$\begin{aligned} \therefore\ \text{Threshold K. E. of alphas} &= -(11/7)\,Q \\ &= -(11/7)\,(-2.85) \\ &= 4.48 \text{ Mev} \end{aligned}$$

Equation (1-2) means that in order for the reaction $Li^7\,(\alpha, n)\,B^{10}$ to take place the incident alpha particles must have a kinetic energy of 4.48 Mev or greater.

An even more useful alpha-neutron reaction is that in which alphas are incident upon beryllium. The resulting reactions are *exothermic:*

$$Be^9 + He^4 \longrightarrow C^{12} + n \tag{1-3}$$

$$Be^9 + He^4 \longrightarrow 3He^4 + n$$

in which the first reaction is more probable and takes place with Q = +5.5 Mev. Being exothermic there is no threshold. However, the Coulomb repulsion of the alpha particles by the beryllium nucleus diminishes the chance of a successful collision by a slow-moving alpha. The net result is that the yield of neutrons from a thin beryllium target (thin to reduce straggling effects from alpha-electron collisions) increases with increasing energy of the incident alpha particles as illustrated in the graph, Figure 1.

* The masses are simplified to the mass-numbers in this calculation, i.e., Li^7, He^4, B^{10} and n^1 are represented by masses 7, 4, 10, 1, respectively. This introduces a negligible error in the computed result.

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