$$\xi = \rho x \qquad \eta = \sigma y \qquad \tau = \sqrt{3g(\rho - \sigma)}\ t. \tag{5}$$

With these new coordinates, the equations (4) and (3) become

$$(1 + \frac{\eta}{\xi})\frac{\dot{\xi}^2}{\xi} + (1 + \frac{\xi}{\eta})\frac{\dot{\eta}^2}{\eta} + (\frac{1}{\eta} + \frac{1}{\xi})\dot{\xi}\dot{\eta} = 1 \tag{6}$$

$$(2 + 2\frac{\eta}{\xi})\ddot{\xi} + (1 + \frac{\xi}{\eta})\ddot{\eta} - \frac{\eta\dot{\xi}^2}{\xi^2} + (\frac{2}{\eta} + \frac{4}{\xi})\dot{\xi}\dot{\eta} - 2\frac{\xi\dot{\eta}^2}{\eta^2} = 1. \tag{7}$$

The dots represent in these equations derivatives with respect to τ. By making use of the similarity properties of these equations, they can be reduced to the first order. The appropriate transformations are the following

$$\left.\begin{aligned} \xi &= e^{2s+2q} & \eta &= e^{2s-2q} \\ 4p &= \frac{\dot{\xi}}{\sqrt{\xi}} + \frac{\dot{\eta}}{\sqrt{\eta}} & r &= \frac{dq}{ds} \end{aligned}\right\} \tag{8}$$

By substitution one obtains the following equation of the first order

$$r\frac{dr}{dq} = (3 + r^2)\,(\mathrm{tgh}\ 2q - \frac{5}{3}r), \tag{9}$$

and also the additional equations

$$\left.\begin{aligned} \dot{s} &= \frac{e^{-s}}{\sqrt{8(3 + r^2)\cosh 2q}}\ ; \qquad \frac{\dot{\xi}}{\sqrt{\xi}} = \frac{e^{q}\,(1 + r)}{\sqrt{2(3 + r^2)\cosh 2q}} \\ \frac{\dot{\eta}}{\sqrt{\eta}} &= \frac{e^{-q}\,(1 - r)}{\sqrt{2(3 + r^2)\cosh 2q}} \end{aligned}\right\} \tag{10}$$

which can be used in passing from the solution of equation (9) to the solution of our physical problem.