TAYLOR INSTABILITY OF INCOMPRESSIBLE LIQUIDS

PART 1. TAYLOR INSTABILITY OF AN INCOMPRESSIBLE LIQUID

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This is an attempt to discuss in a very simplified form the problem of the growth of an initial ripple on the surface of an incompressible liquid in presence of an acceleration, g, directed from the outside into the liquid.

The model is that of a heavy liquid occupying at $t = 0$ the half space above the plane $z = 0$. It is well known that this is a state of unstable equilibrium. Any tiny ripple on the surface at the initial time grows in amplitude, first exponentially and later, when its amplitude has become comparable to the wave length, by a more complicated law.

The case will be considered that there is initially a small amplitude sinusoidal ripple of wave length λ. In a first phase this amplitude will increase exponentially like

$$\exp\left(\sqrt{\frac{2\pi g}{\lambda}}\, t\right) \tag{1}$$

This exponential law, however, will break down when the amplitude has become comparable to $\lambda\!\!\!^{-} = \lambda/2\pi$. We propose to discuss what happens in the subsequent phase.

This will be done by grossly schematizing the shape of the wave as indicated in Fig. 1.

Instead of a wave profile like the curve, a profile like ABCDEFGHIJ will be assumed.

It is clear from the symmetry of the problem that the points at the maximum and the minimum of the wave move in vertical directions. In Fig. 2 a half wave, from a maximum to the successive minimum is represented with the notations adopted. OO' is the initial level of the liquid. On account of the incompressibility the amount of liquid below the plane OO', namely CO'DE